

Implications for quarkonia

Quarkonia dissolve from:

Cold Nuclear Matter

Debye screening

Landau damping...

Today: *very* near T_c , $T_c < T < 1.2 T_c$, qualitatively *new* region.

Not just “a” screening mass. Higgs effect gives “split” masses:
some heavy, some light.

Propagation of color fields *dynamically* suppressed

Effects on Debye screening, Landau damping, for quarkonia?

An effective (matrix) model for deconfinement

1. Lattice: $SU(N)$ gauge theories, *without* quarks. *Most* quantities scale with N .
Not just the pressure: Polyakov loop, correlation functions, interface tensions

2. Matrix model: *simple* mean field model, valid in large N expansion

Fit pressure, including latent heat, for *all* N , with two (N -dependent) parameters

Good agreement with interface tensions, both order-order and order-disorder

Problem: sharp *disagreement* with the (renormalized) Polyakov loop - ?

Qualitatively *new*: adjoint Higgs phase, pattern of *split* masses, for $T < 1.2 T_c$.

Unexpected punchline: transition region *very narrow*, $< 1.2 T_c$!

Dumitru, Guo, Hidaka, Korthals-Altes, & RDP, arXiv:1011.3820 + 11... DGHKP
Generalization of Meisinger, Miller, Ogilvie ph/0108009, MMO

Also: ...RDP, ph/0608242; Y. Hidaka & RDP, 0803.0453, 0906.1751, 0907.4609, 0912.0940.

What the lattice tells us

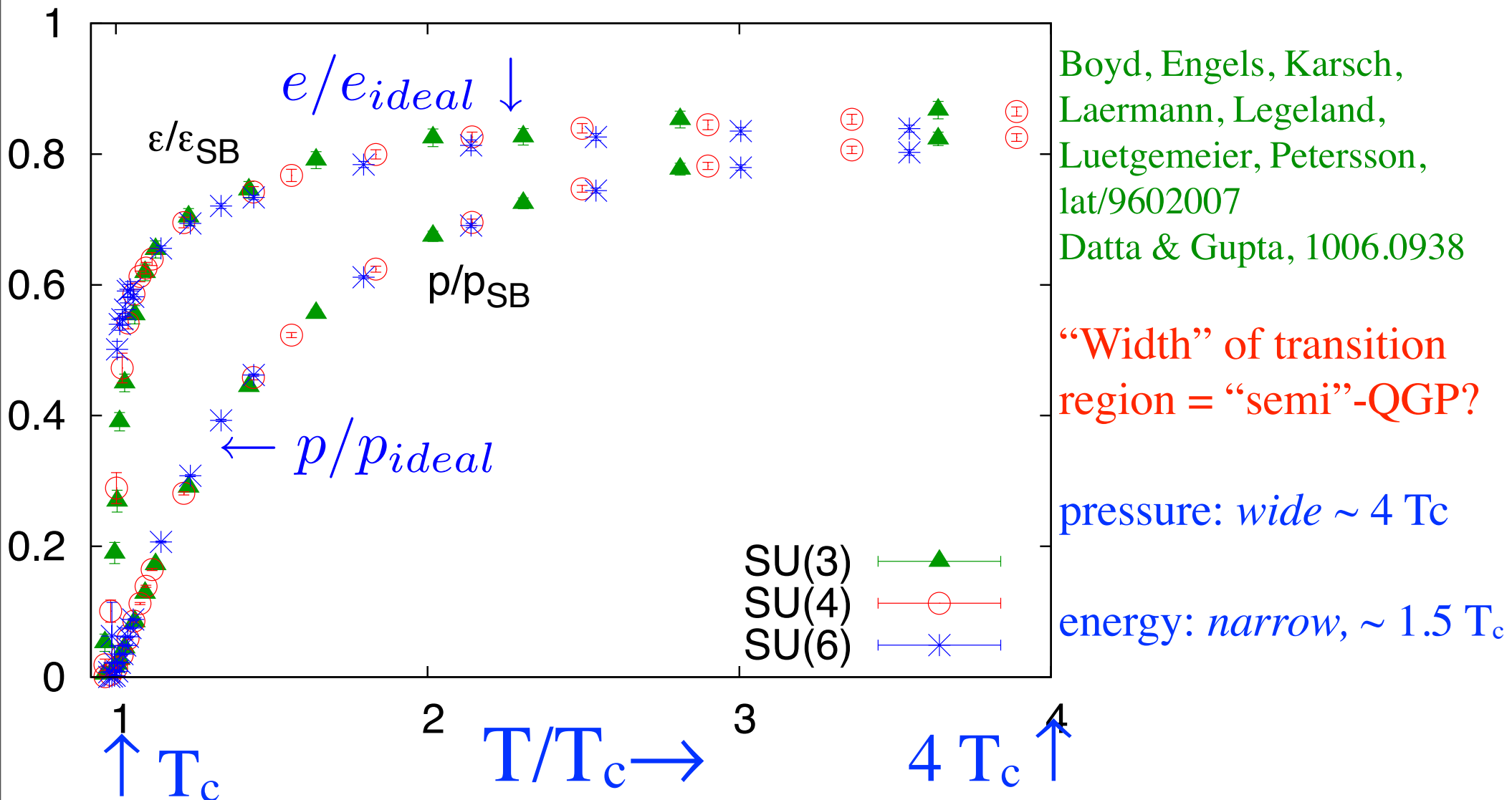
Its *not* just the pressure...

Lattice: SU(N) thermodynamics, $T_c \rightarrow 4 T_c$

SU(N) gauge theories *without* quarks, temperature $T \neq 0$

Scaled by ideal gas, energy and pressure *approximately* independent of N.

e and $p \approx 0$ below T_c : $\sim N^2 - 1$ gluons above T_c , vs ~ 1 hadrons below.

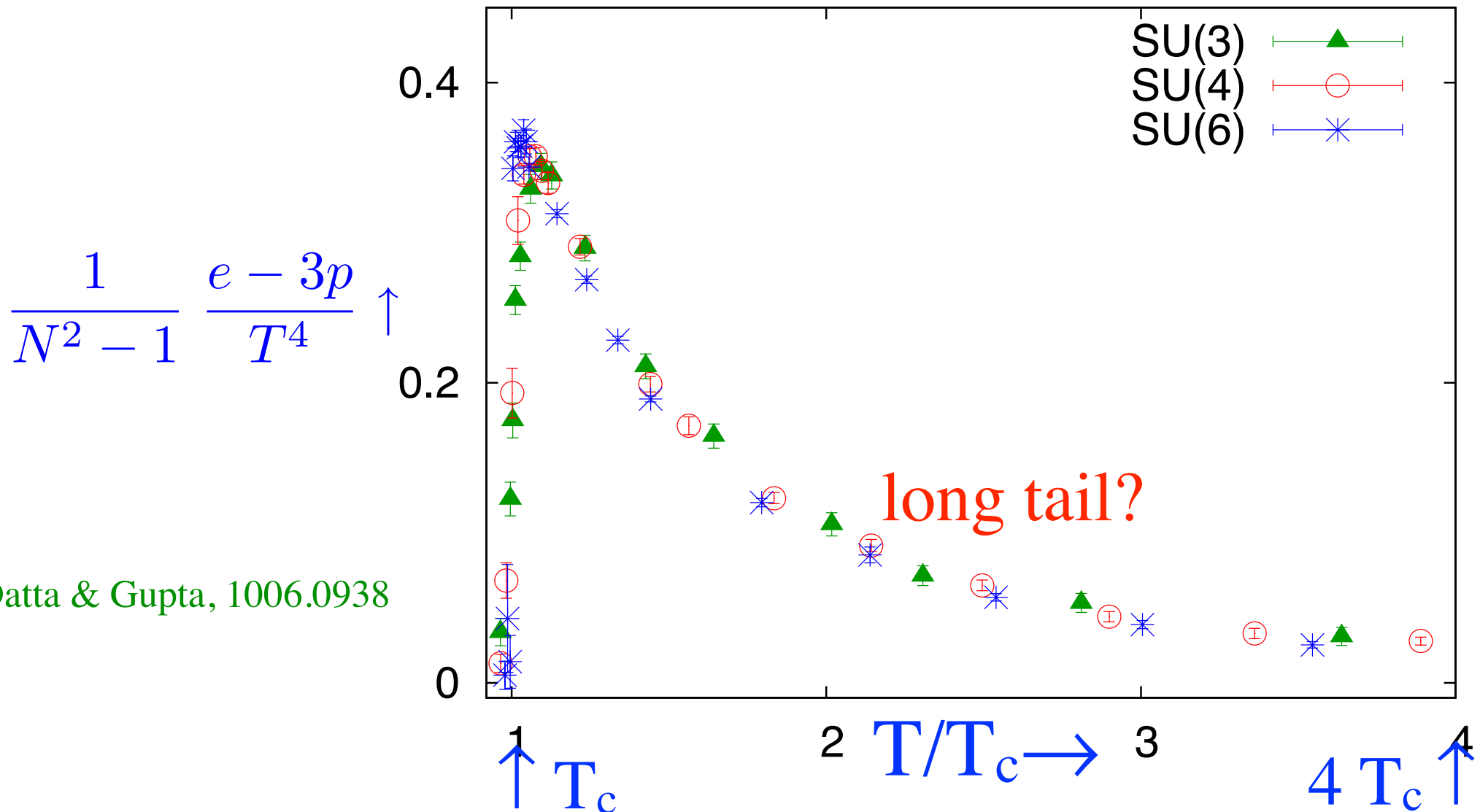


Lattice: peak in conformal anomaly

For SU(N), “peak” in $e-3p/T^4$ just above T_c . *Approximately* uniform in N.

Not near T_c : transition *2nd* order for $N = 2$, *1st* order for *all* $N \geq 3$

$N=3$: *weakly* 1st order. $N = \infty$: *strongly* 1st order (latent heat $\sim N^2$)



Datta & Gupta, 1006.0938

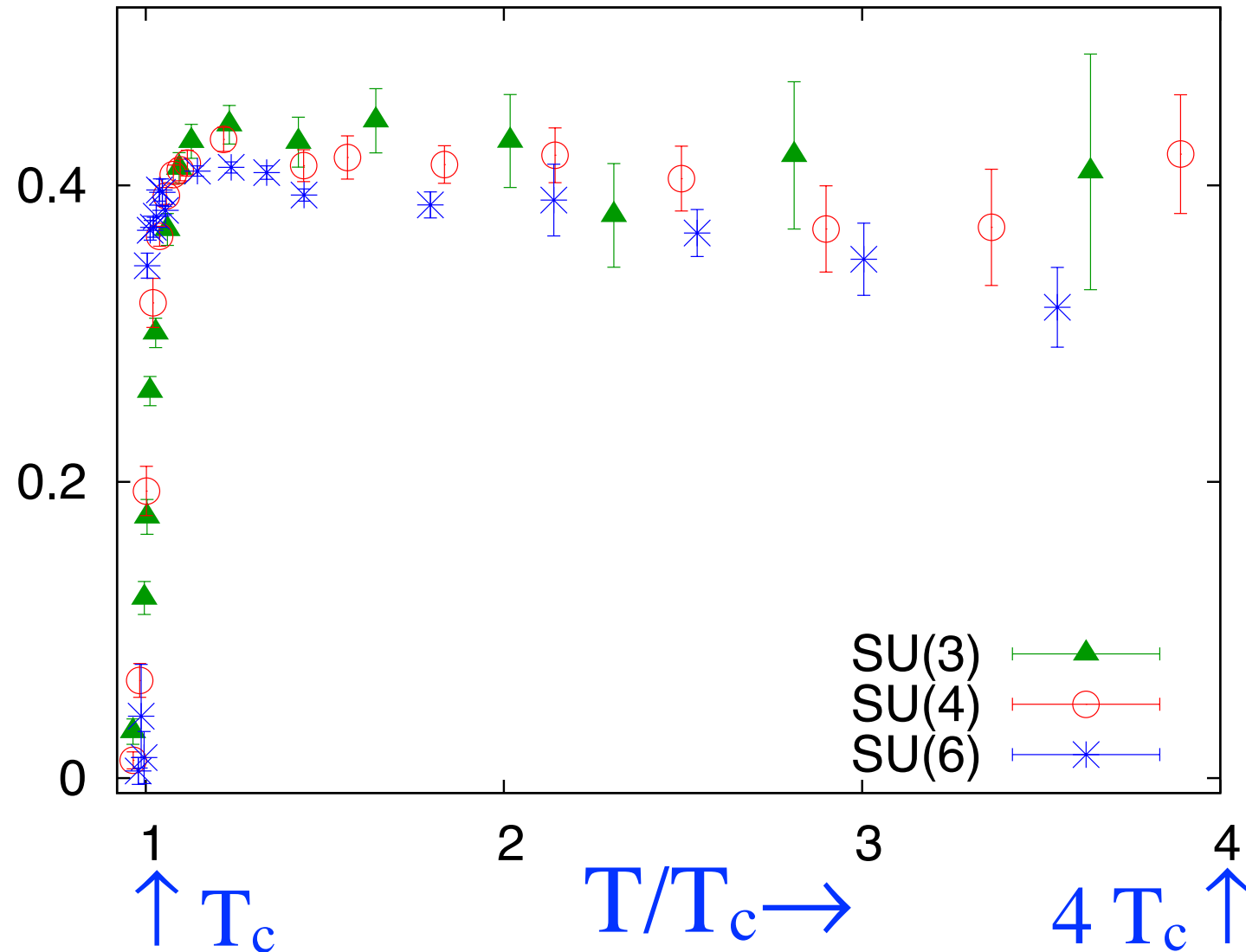
Lattice: scaling of the conformal anomaly

Scaling: $(e-3p)/T^2$ approximately constant near T_c : MMO '01; RDP, ph/0608242

Only true near T_c ; eventually, $(e-3p)/T^4 \sim g^4(T)$

$$\frac{1}{N^2 - 1} \frac{e - 3p}{T^2 T_c^2} \uparrow$$

Datta & Gupta, 1006.0938



Lattice: *precise* scaling of the conformal anomaly

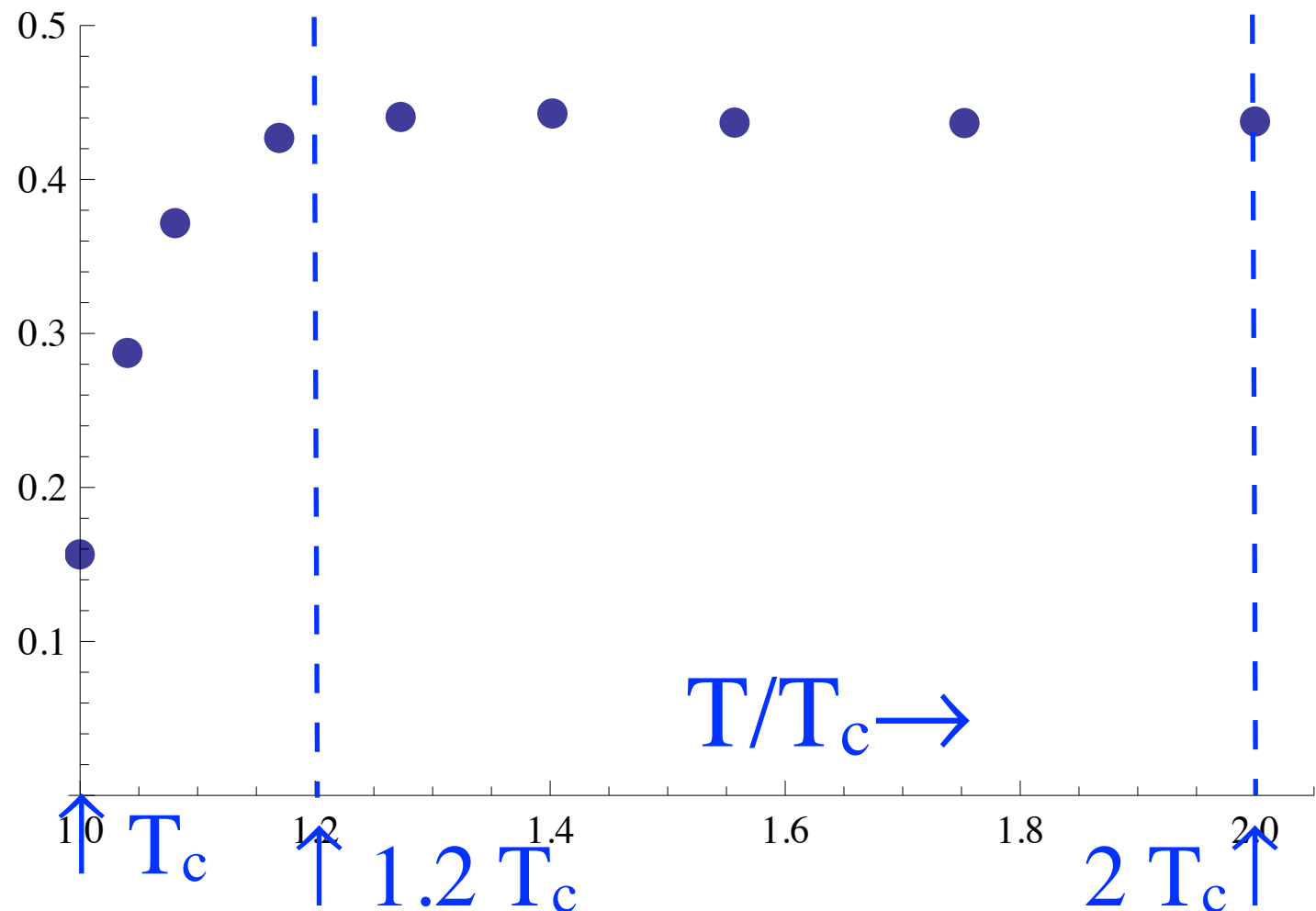
Lattice: WHOT. Change # time steps at fixed lattice scale. Higher precision, $\pm 1\%$

$$T : 1.2 \rightarrow 2 T_c : \frac{e - 3p}{T^2} \approx (543 \text{ MeV})^2 \pm 1\%$$

$$p(T) \approx \# T^2 (T^2 - c T_c^2), \quad c = 1.00 \pm .01$$

$$\frac{1}{8} \frac{e - 3p}{T^2 T_c^2} \uparrow$$

Umeda, Ejiri, Aoki, Hatusda,
Kanaya, Maezawa, Ohno,
0809.2842

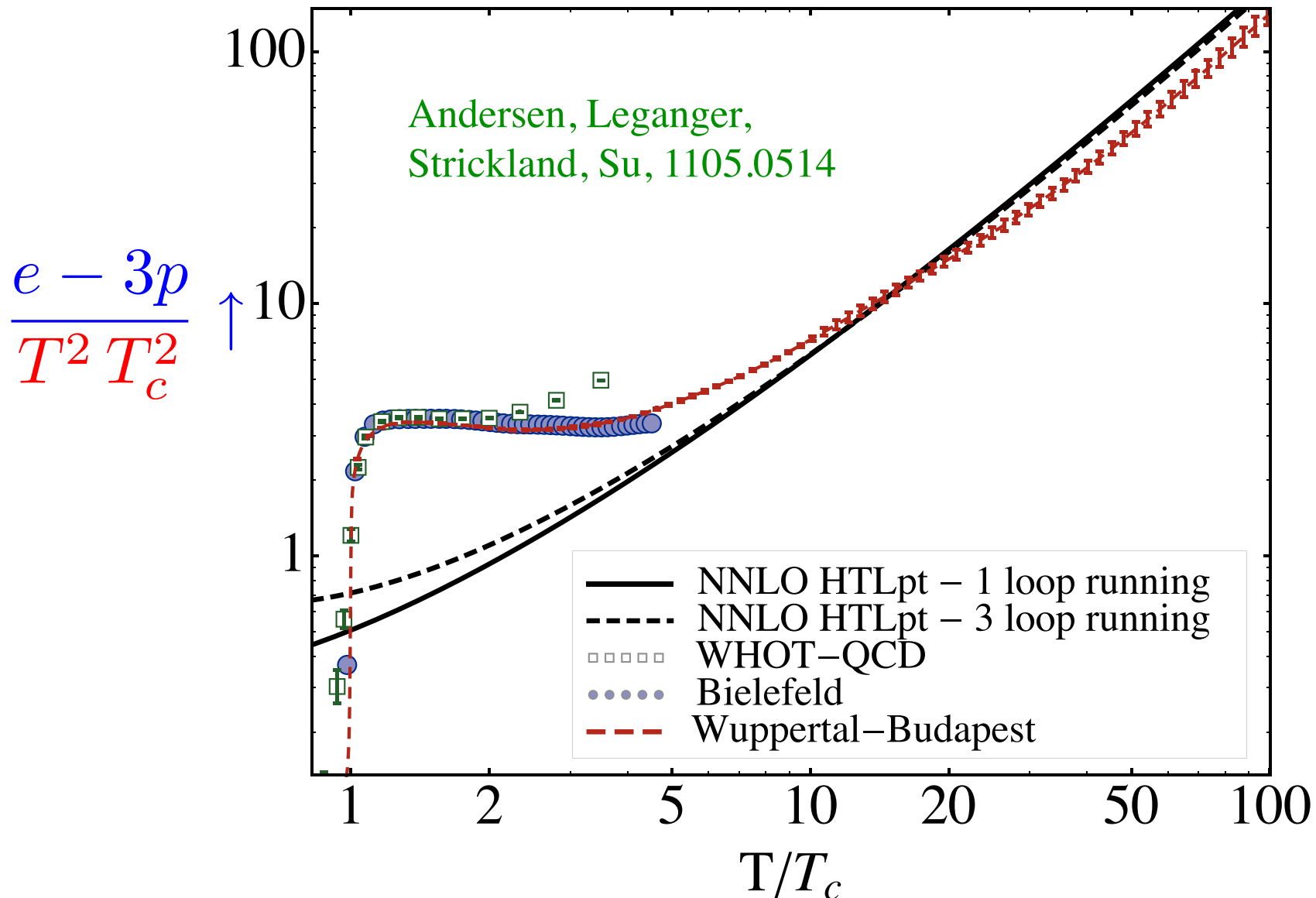


Lattice vs resummed perturbation theory

HTL resummed perturbation thy., next to next to leading order, works down to $\sim 8 T_c$?

Assume that the QCD coupling runs like $\alpha(2\pi T)$: Braaten & Nieto, hep-ph/9501375

Coupling is *moderate* even at T_c , $\alpha(2\pi T_c) \sim 0.3$,



“Hidden” Z(2) spins in SU(2)

Consider $U_c = \text{constant}$ gauge transf.

Two colors: $\det U_c = (-)^2 = 1$, so $U_c \in \text{SU}(2)$

$$U_c = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -\mathbf{1}$$

U_c very special: $\sim \mathbf{1}$, so gluons *invariant*:

$$A_\mu \rightarrow U_c^\dagger A_\mu U_c = + A_\mu$$

If gluons invariant, does U_c matter?

At temperature $T \neq 0$, Wilson line changes:

$$\mathbf{L} = \mathcal{P} e^{ig \int_0^{1/T} A_0 d\tau} \rightarrow -\mathbf{L}$$

τ = imaginary time: $0 \rightarrow 1/T$:

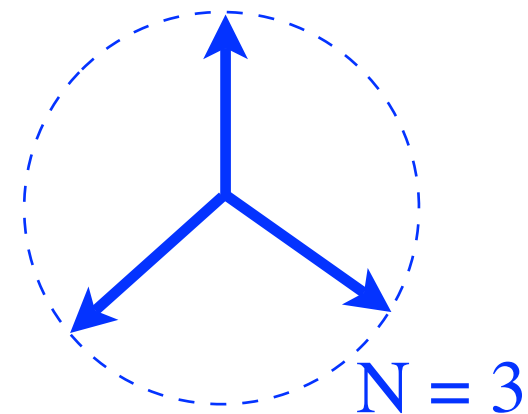
aperiodic gauge transf., $U(1/T) = U_c U(0)$.

\mathbf{L} propagator for “test” quark, with color electric charge.

SU(3): 3rd root of unity, $j=0,1,2$

SU(N): Nth root, Z(N) symmetry.

$$U_c = e^{2\pi i j/3} \mathbf{1}$$



Z(N) spins of ‘t Hooft, *without* quarks

Quarks act like background Z(N) field, *break* Z(N) symmetry

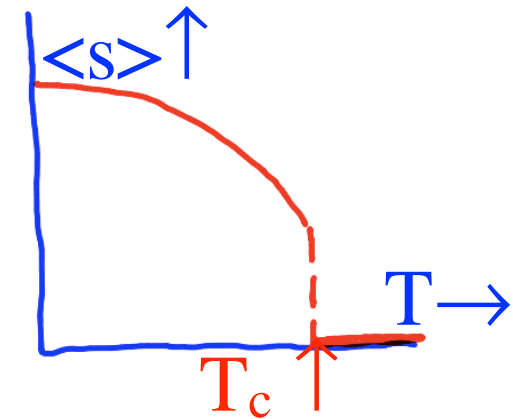
$$\psi \rightarrow -\psi$$

Usual Z(2) spins vs Polyakov Loop

Ordinary spins, s : symmetry broken at low T ,
restored for $T > T_c$. First order transition:

$\mathbf{L} = \text{SU}(N)$ matrix. Trace = Polyakov loop, l :

$$\ell = \frac{1}{N} \text{tr } \mathbf{L}$$

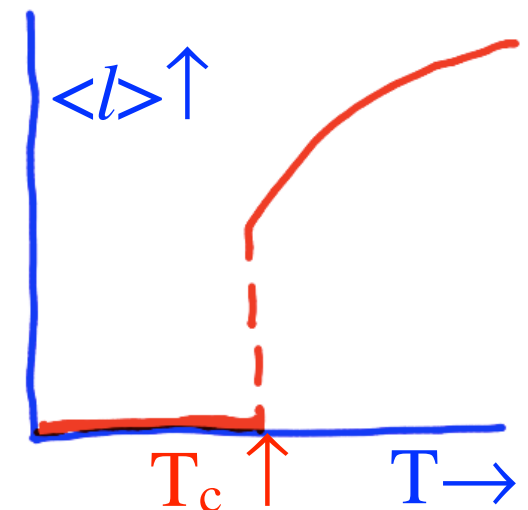


$\langle l \rangle$ gauge invariant, measures color ionization:

$$\langle \ell \rangle \sim e^{-F_{\text{test qk}}/T}$$

Color is not ionized in confined phase, so
confinement $\Rightarrow \langle l \rangle = 0$: $Z(N)$ symmetric phase.

Color ionized above T_c , so
 $\langle l \rangle \neq 0$, $Z(N)$ broken, above T_c .



Svetitsky and Yaffe '80: SU(3) 1st order because of $Z(3)$ symmetry:

Eff. Lag. of loops has cubic terms, $l^3 + (l^*)^3$.

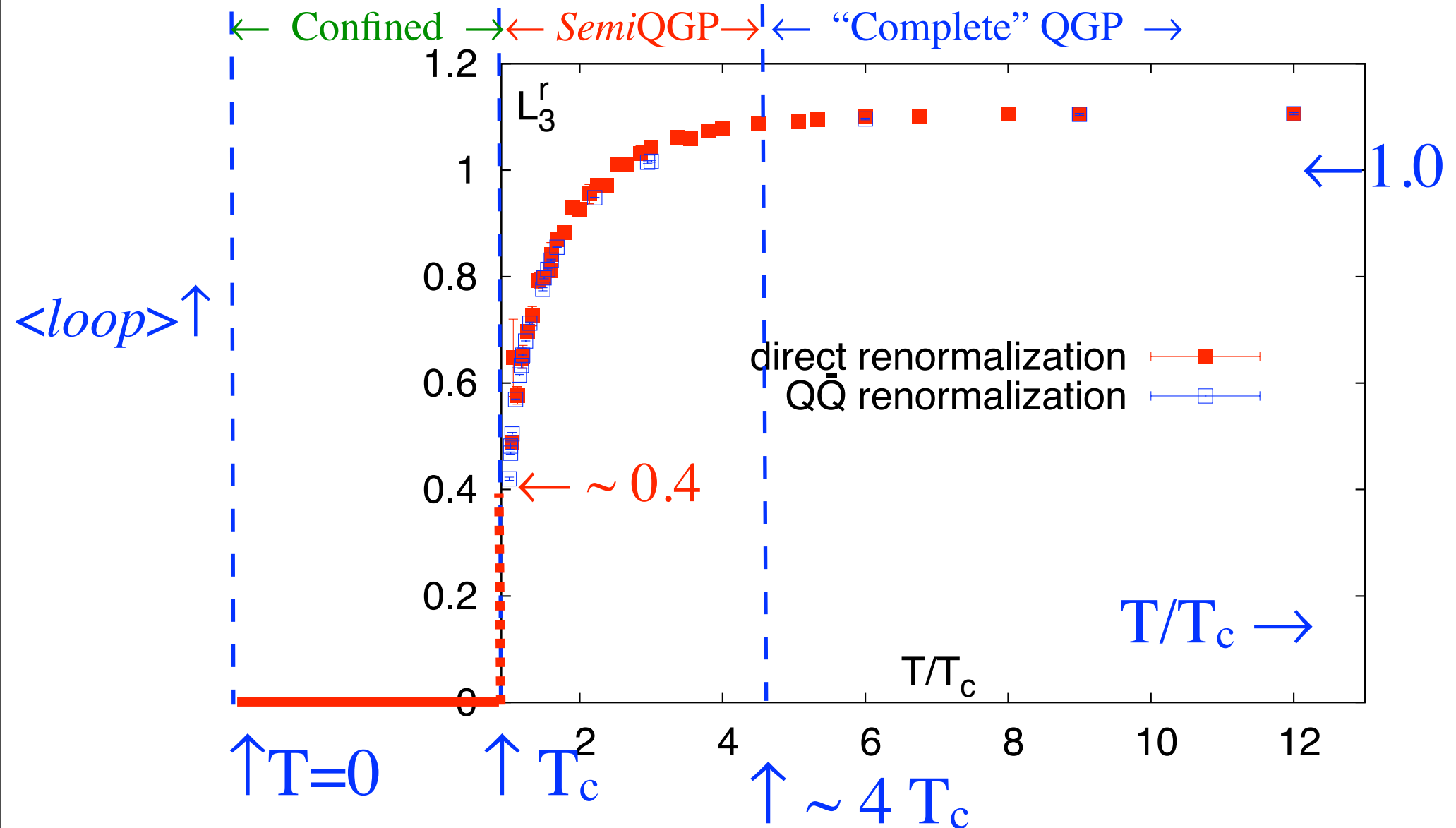
Does *not* apply for $N > 3$. So why is deconfinement 1st order for *all* $N \geq 3$?

Polyakov Loop from Lattice: pure Glue, no Quarks

Lattice: (*renormalized*) Polyakov loop. Strict order parameter

Three colors: Gupta, Hubner, Kaczmarek, 0711.2251.

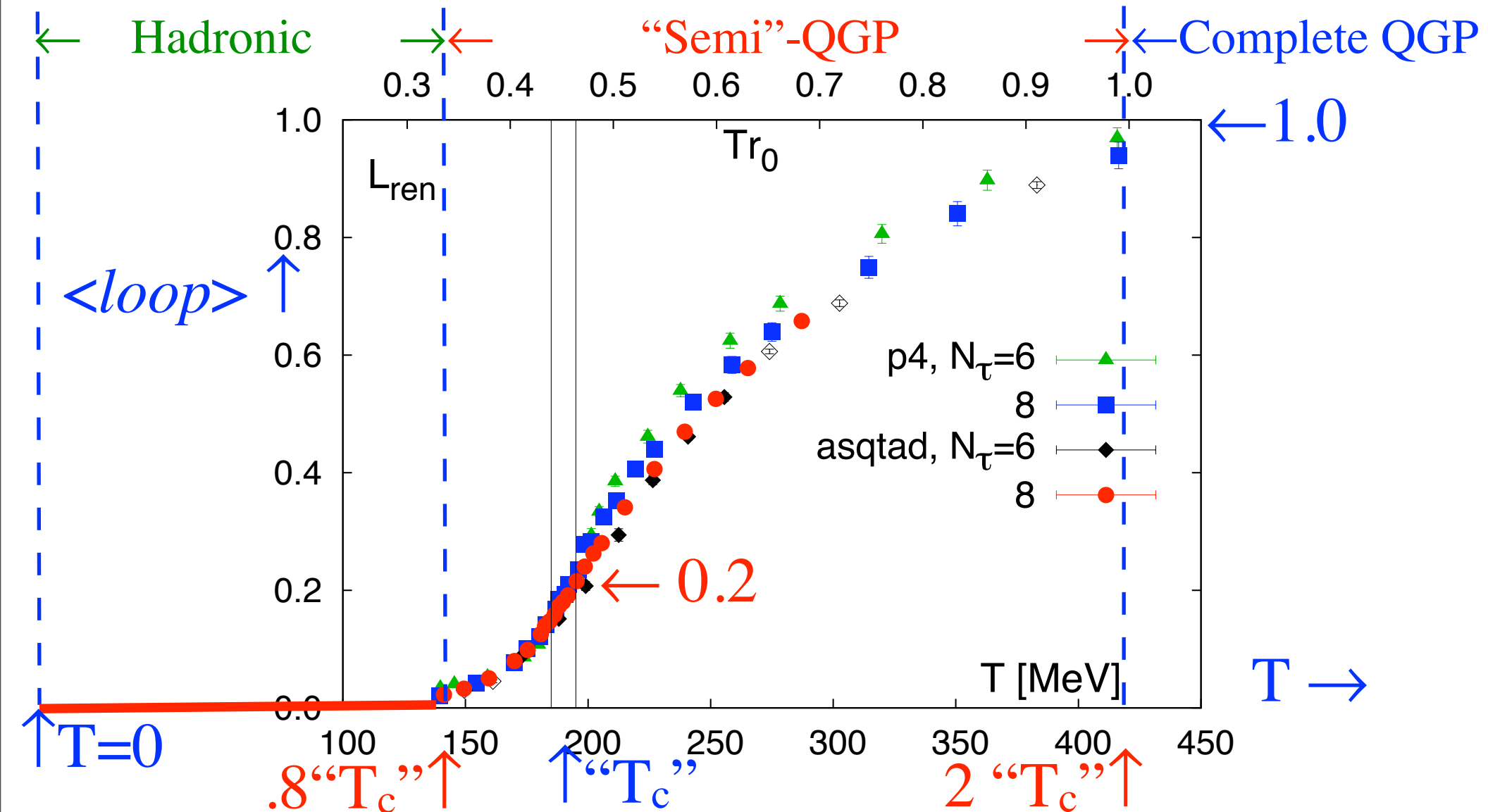
Suggests transition region, “semi” QGP, is wide, to $\sim 4 T_c$, like pressure.



Polyakov Loop from Lattice: Glue plus Quarks, “ T_c ”

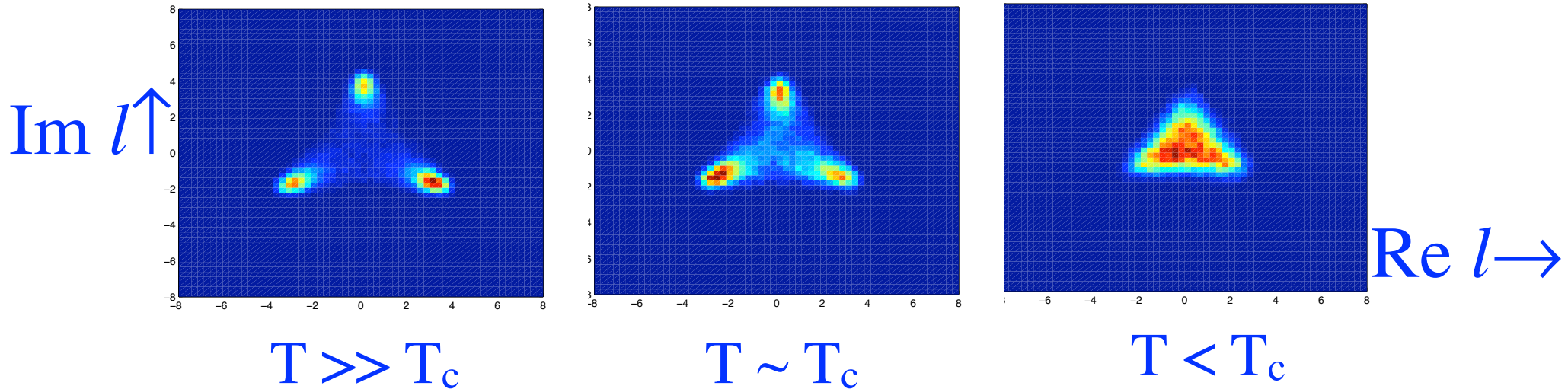
Quarks \sim background $Z(3)$ field. Lattice: Bazavov et al, 0903.4379.

3 quark flavors: *weak* $Z(3)$ field, does *not* wash out approximate $Z(3)$ symmetry.

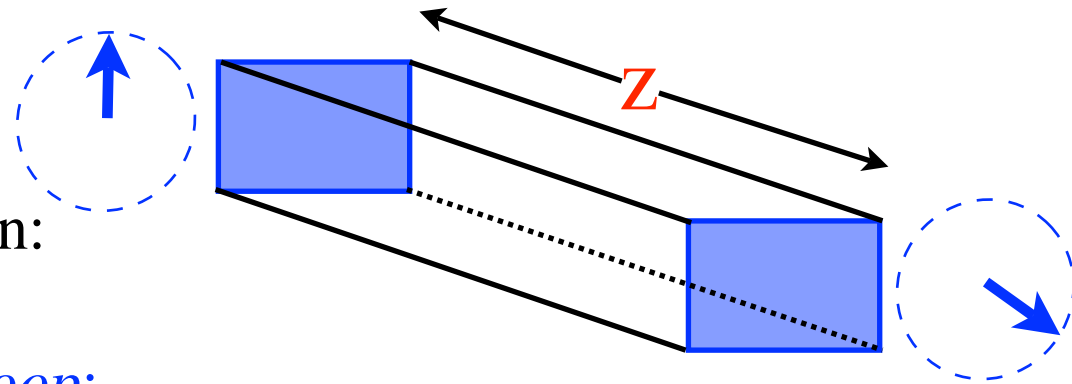


Interface tensions: order-order & order-disorder

Lattice, A. Kurkela, unpub.'d: 3 colors, loop l complex. Distribution of loop shows $Z(3)$ symmetry



Interface tension: box long in z .
Each end: distinct but *degenerate* vacua.
Interface forms, action \sim interface tension:



$T > T_c$: order-order interface = 't Hooft loop:

measures response to *magnetic* charge

Korthals-Altes, Kovner, & Stephanov, hep-ph/9909516

$$Z \sim e^{-\sigma_{int} V_{tr}}$$

Also: if trans. 1st order, order-*disorder* interface at T_c .

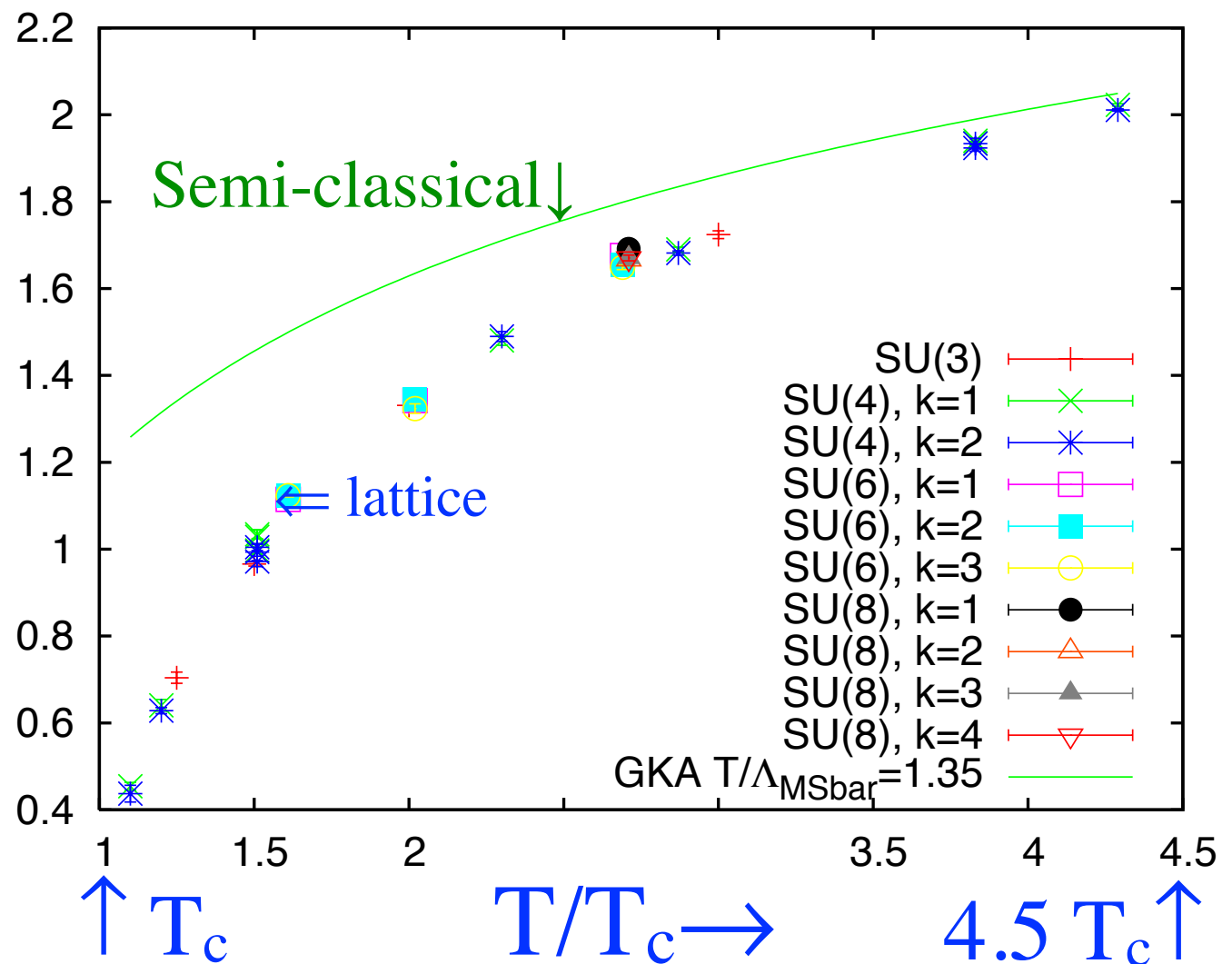
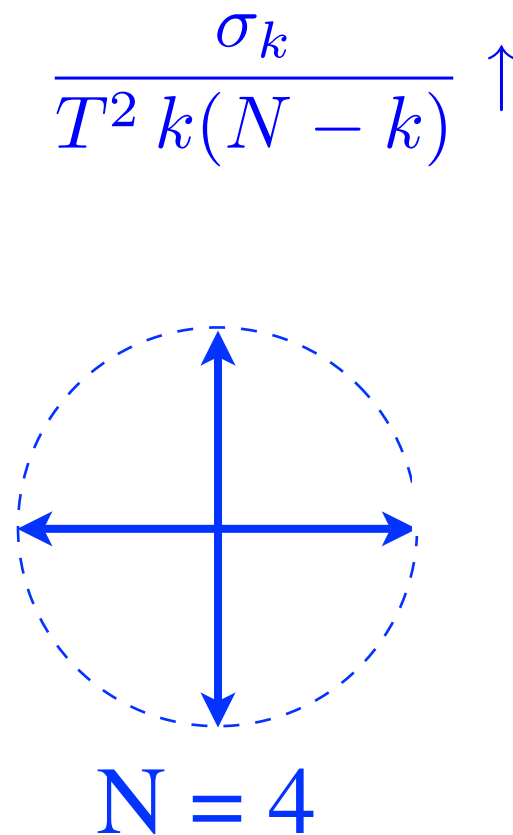
Lattice: order-order interface tensions σ

Lattice: de Forcrand & Noth, lat/0510081. $\sigma \sim$ universal with N

Semi-classical σ : Giovanengelli & Korthals-Altes ph/0102022; /0212298; /0412322: *GKA '04*

Above $4 T_c$, semi-class $\sigma \sim$ lattice. Below $4 T_c$, lattice $\sigma \ll$ semi-classical σ .

Even so, when $N > 3$, *all* tensions satisfy “Casimir scaling” at any T



Lattice: how does A_0 mass change as $T \rightarrow T_c$?

Gauge invariant: 2 pt function of loops:

$$\langle \text{tr } \mathbf{L}^\dagger(x) \text{tr } \mathbf{L}(0) \rangle \sim e^{-\mu x} / x^d$$

μ/T decreases as $T \rightarrow T_c$

Kaczmarek, Karsch, Laermann,
Lutgemeier lat/9908010

Gauge dependent: singlet potential

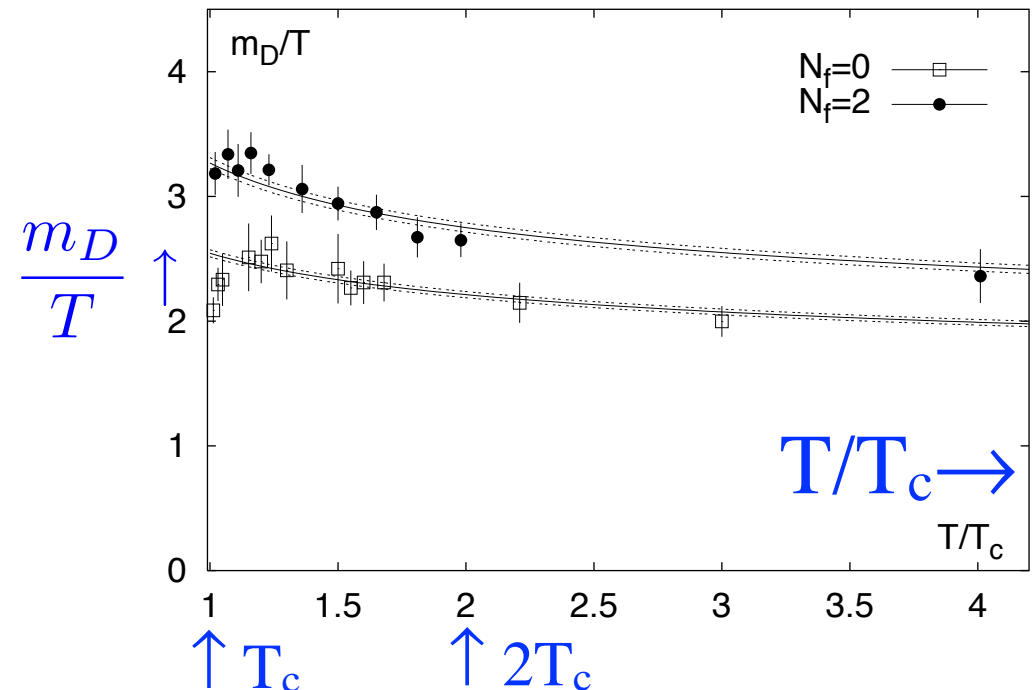
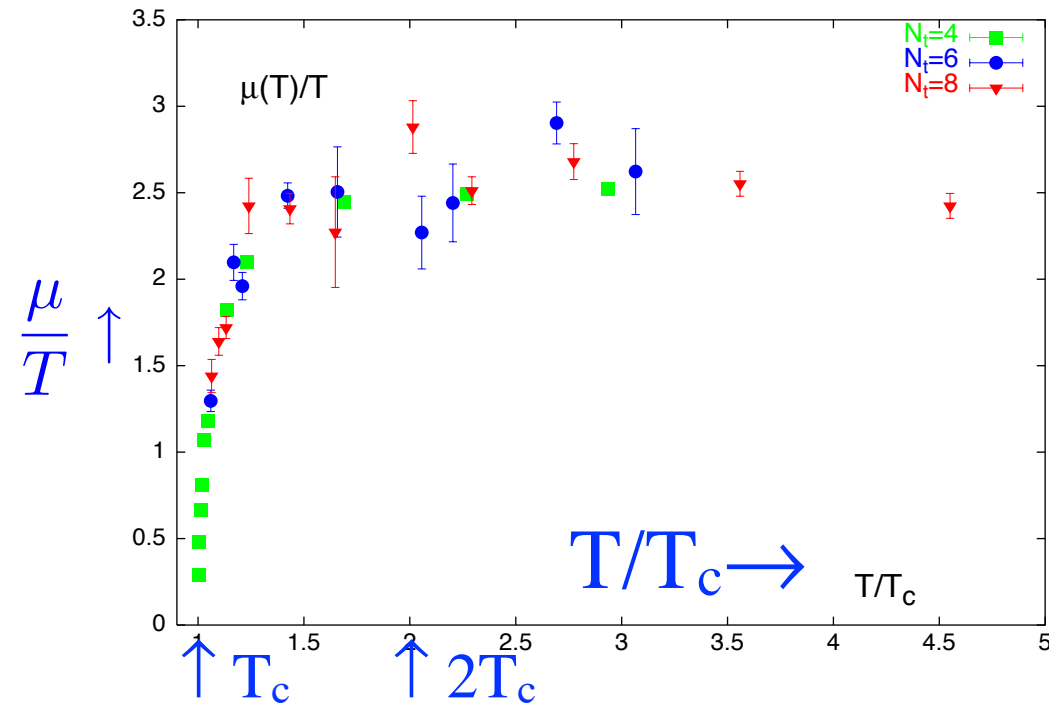
$$\langle \text{tr } (\mathbf{L}^\dagger(x) \mathbf{L}(0)) \rangle \sim e^{-m_D x} / x$$

m_D/T increases as $T \rightarrow T_c$

Which way do masses go as $T \rightarrow T_c$?

Both change below $\sim 1.5 T_c$.

Cucchieri, Karsch, Petreczky lat/0103009,
Kaczmarek, Zantow lat/0503017



The competition

Models for the semi-QGP, T_c to $4 T_c$

1. Massive gluons: Peshier, Kampfer, Pavlenko, Soff '96...Castorina, Miller, Satz 1101.1255
Castorina, Greco, Jaccarino, Zappala 1105.5902

Mass decreases pressure, so adjust $m(T)$ to fit $p(T)$. Simple model.

Gluons *very* massive near T_c .

$$p(T) = \# T^4 - m^2 T^2 + \dots$$

2. Polyakov loops: Fukushima ph/0310121...Hell, Kashiwa, Weise 1104.0572

Effective potential of Polyakov loops.

Potential has 5 parameters, *most* ungainly

With quarks, go from $\mu = 0$ to $\mu \neq 0$, at $T \neq 0$

$$V_{eff}(T) \sim m^2 \ell^* \ell + T \log f(\ell^* \ell)$$

$$m^2 = T^4 \sum_{i=0}^3 a_i (T_c/T)^i$$

3. AdS/CFT: Gubser, Nellore 0804.0434...Gursoy, Kiritsis, Mazzanti, Nitti, 0903.2859

Add potential for dilaton, ϕ , to fit pressure.

Only infinite N. Relatively simple potential,

$$V(\phi) \sim \cosh(\gamma\phi) + b\phi^2$$

All of these models fit *only* the pressure, and *not* interface tensions.

Masses: near T_c , massive gluons heavy, Polyakov loops light.

Matrix model: two colors

Simple approximation: expand about *constant*, diagonal A_0

Two colors: transition 2nd order, vs 1st for $N \geq 3$

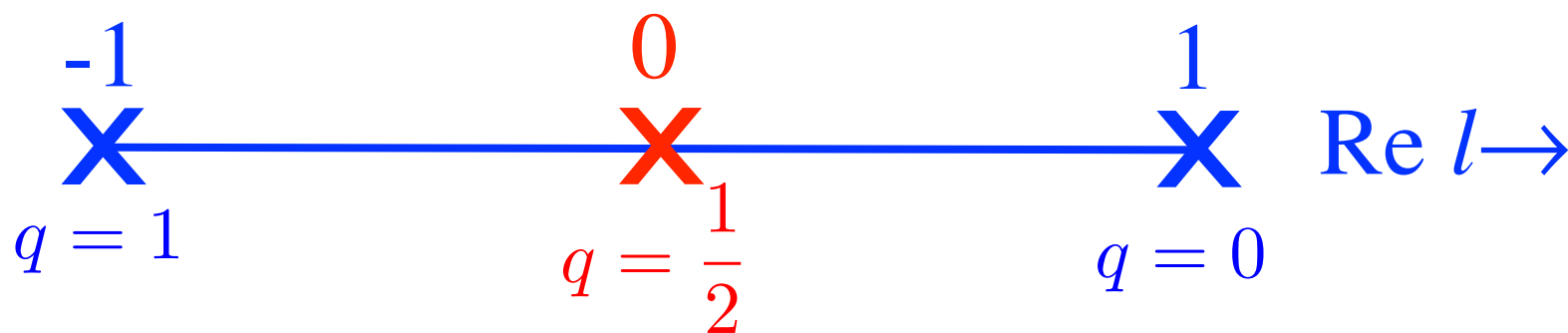
Implicitly: using large N expansion at $N = 2$ (!)

Matrix model: SU(2)

Simplest possible approx.: model *constant* gauge transf.'s with *constant* $A_0 \sim \sigma_3$:

$$A_0 = \frac{\pi T}{g} q \sigma_3, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \mathbf{L}(q) = \begin{pmatrix} e^{i\pi q} & 0 \\ 0 & e^{-i\pi q} \end{pmatrix}$$

Loop l real. $Z(2)$ degenerate vacua $q = 0$ and 1 : $\ell = \cos(\pi q)$



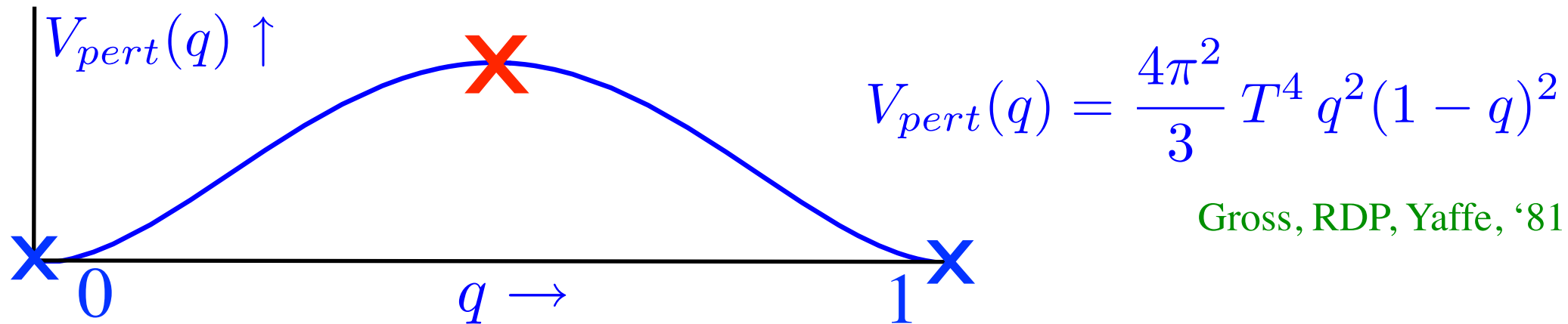
Confined vacuum, \mathbf{L}_c , *halfway* in between:
 $q = 1/2, l = 0$.

$$\mathbf{L}_c = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

Classically, no potential for q .

Potential for q , interface tension

Computing to one loop order about *background* A_0 generates a potential for q :



Use $V_{pert}(q)$ to compute σ : Bhattacharya, Gocksch, Korthals-Altes, RDP, ph/9205231.

$$V_{tot}(q) = \frac{2\pi^2 T^2}{g^2} \left(\frac{dq}{dz} \right)^2 + V_{pert}(q) \quad \Rightarrow \quad \sigma = \frac{4\pi^2}{3\sqrt{6}} \frac{T^2}{\sqrt{g^2}}$$

Computation semi-classical: $A_0 \sim 1/g$, so classical action $\sim 1/g^2$. But V_{pert} only ~ 1

Balancing the two gives $\sigma \sim 1/\sqrt{g^2}$ (and *not* $1/g^2$). Interface *large* in z , $\sim 1/\sqrt{g^2} T$.

Justifies expansion about constant A_0 . GKA '04: corrections to $\sigma \sim g^3$. g^4 ?

Symmetries of the q 's

Wilson line \mathbf{L} *not* gauge invariant, $\mathbf{L} \rightarrow \Omega^\dagger \mathbf{L} \Omega$. Its eigenvalues, $e^{\pm i \pi q}$, are.

The *ordering* of the eigenvalues of \mathbf{L} is immaterial. Symmetries:

In all cases: $q \rightarrow q + 2$: q angular variable.
Valid with quarks.

$$\mathbf{L}(q) = \begin{pmatrix} e^{i\pi q} & 0 \\ 0 & e^{-i\pi q} \end{pmatrix}$$

Pure glue: also, $q \rightarrow q + 1$: $Z(2)$ transf., $\mathbf{L} \rightarrow -\mathbf{L}$

For pure glue, can restrict q : $0 \rightarrow 1$.

$$\mathbf{L}(1 - q) = - \begin{pmatrix} e^{-i\pi q} & 0 \\ 0 & e^{i\pi q} \end{pmatrix}$$

Then $Z(2)$ transf. $q \rightarrow 1 - q$:

$Z(2)$ transf., *plus* exchange of eigenvalues

Hence for pure glue, any potential of q must be invariant under $q \rightarrow 1 - q$

Potentials for the q 's

Consider expansion about perturbative vacuum. GKA '04: to $\sim g^3$.

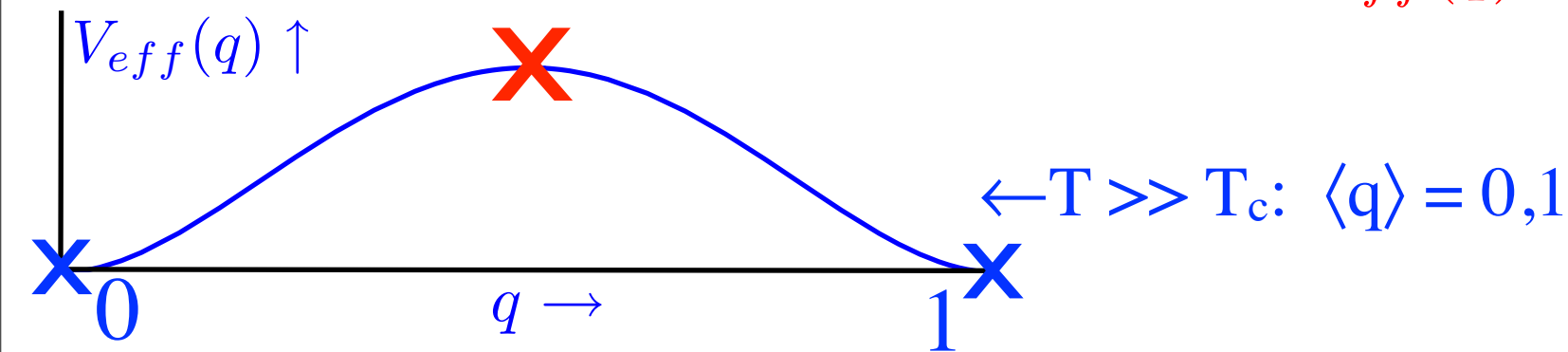
To 2 loop order, find terms $V_{\text{pert}} \sim g^2 T^4 q(1-q)$. Destabilize pert. vacuum?

Absorbed into 1 loop corrected eigenvalues of \mathbf{L} , $e^{\pm i \pi q_{\text{ren}}}$

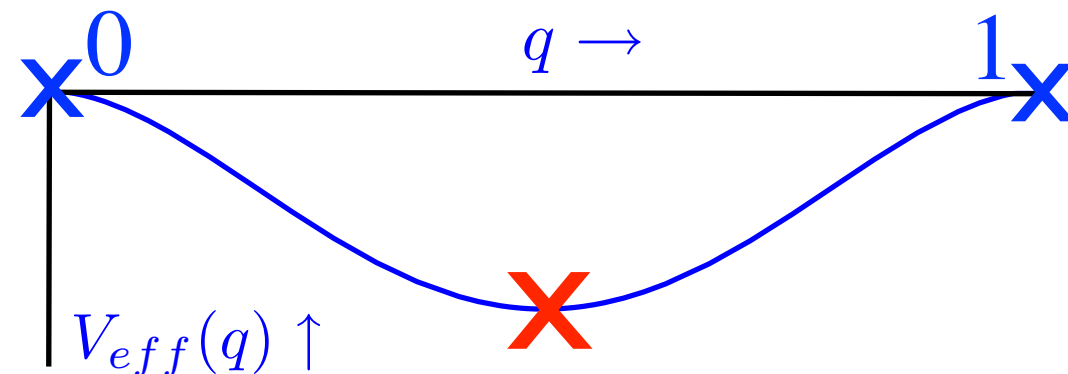
Perturbatively, $q_{\text{ren}} = 0$ order by order in g . Gocksch & RDP, ph/9302233

Add *non-pert.* terms, by *hand*, to generate $\langle q \rangle \neq 0$:

$$V_{\text{eff}}(q) = V_{\text{pert}} + V_{\text{non}}$$



$T < T_c: \langle q \rangle = 1/2 \rightarrow$



Possible “phases” and transitions

Three possible “phases”:

$\langle q \rangle = 0, 1: \langle l \rangle = \pm 1$: “Complete” QGP: usual perturbation theory. $T \gg T_c$.

$0 < \langle q \rangle < 1/2: \langle l \rangle < 1$: “semi”-QGP. Adjoint Higgs phase for A_0 . $x T_c > T > T_c \ x?$

$\langle q \rangle = 1/2: \langle l \rangle = 0$: confined phase. $T < T_c$

Lattice: evidence for semi-QGP, where $\langle l \rangle < 1$:

not just direct transition from complete QGP, to confined phase.

Possible to have a phase transition from a phase symmetric in q , $\langle q \rangle = 0$, to Higgs phase, $\langle q \rangle \neq 0$, at some temperature *above* T_c .

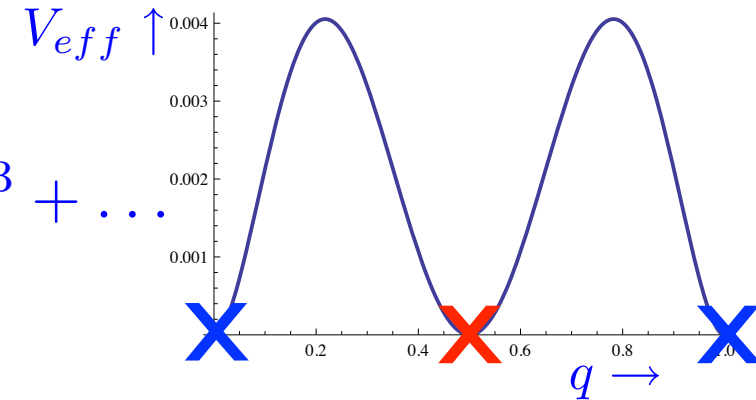
(Since adjoint Higgs phase, though, need *not* have transition.)

Strongly constrains possible non-perturbative terms, $V_{\text{non}}(q)$.

Getting three “phases”, one transition

Simple guess: $V_{\text{non}} \sim \text{loop}^2$,

$$V_{\text{eff}} \sim \frac{a}{\pi^2} (\ell^2 - 1) + q^2(1 - q)^2 \sim q^2(1 - a) - 2q^3 + \dots$$



Transition at “a”=1/16, but 1st order, *directly* from complete QGP, to confinement
No semi-QGP? Also, transition to confined phase should be 2nd, not 1st, order.

If $V_{\text{non}}(q) \sim q^2$ at small q , usually: direct 1st order transition, with *no* semi-QGP,
or: 2nd order transition at a $T > T_c$, from complete to semi-QGP.

Easy to avoid, *if* $V_{\text{non}}(q) \sim q$ for small q . Then $\langle q \rangle \neq 0$ at all T .

Imposing the symmetry of $q \leftrightarrow 1 - q$, $V_{\text{non}}(q)$ *must* include

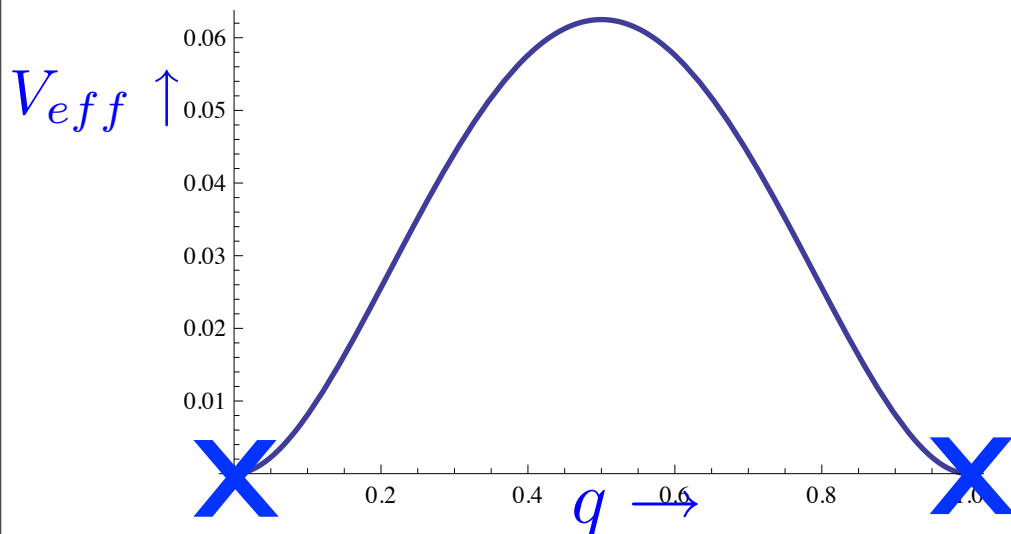
$$V_{\text{non}}(q) \sim q(1 - q)$$

Cartoons of deconfinement

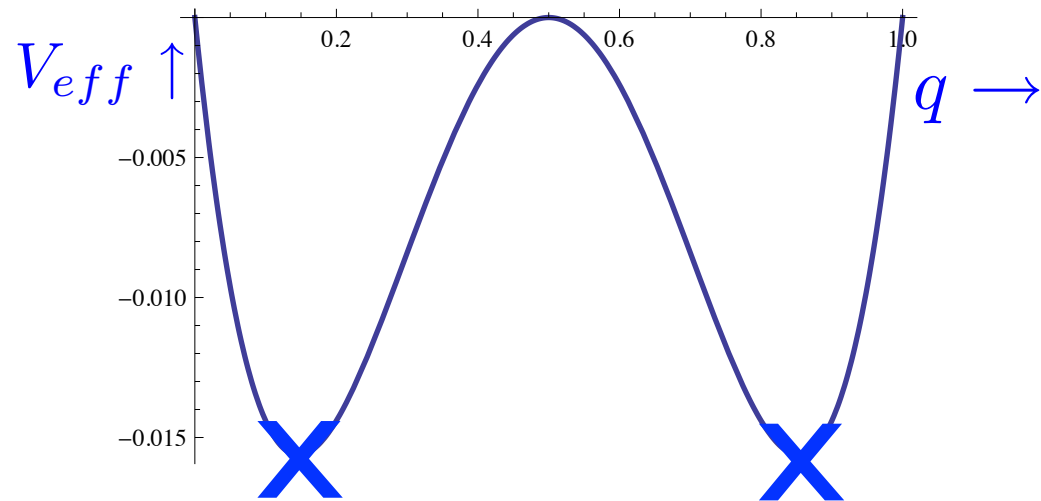
Consider:

$$V_{eff} = q^2(1 - q)^2 - a q(1 - q), \quad a \sim T_c^2 / T^2$$

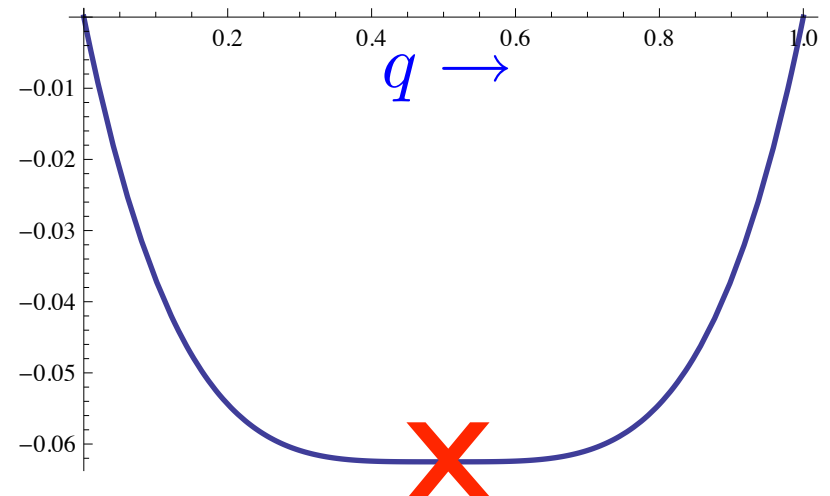
↓ $a = 0$: complete QGP



↓ $a = 1/4$: semi QGP



$a = 1/2$: $T_c \Rightarrow$
Stable vacuum at $q = 1/2$
Transition *second* order



0-parameter matrix model, $N = 2$

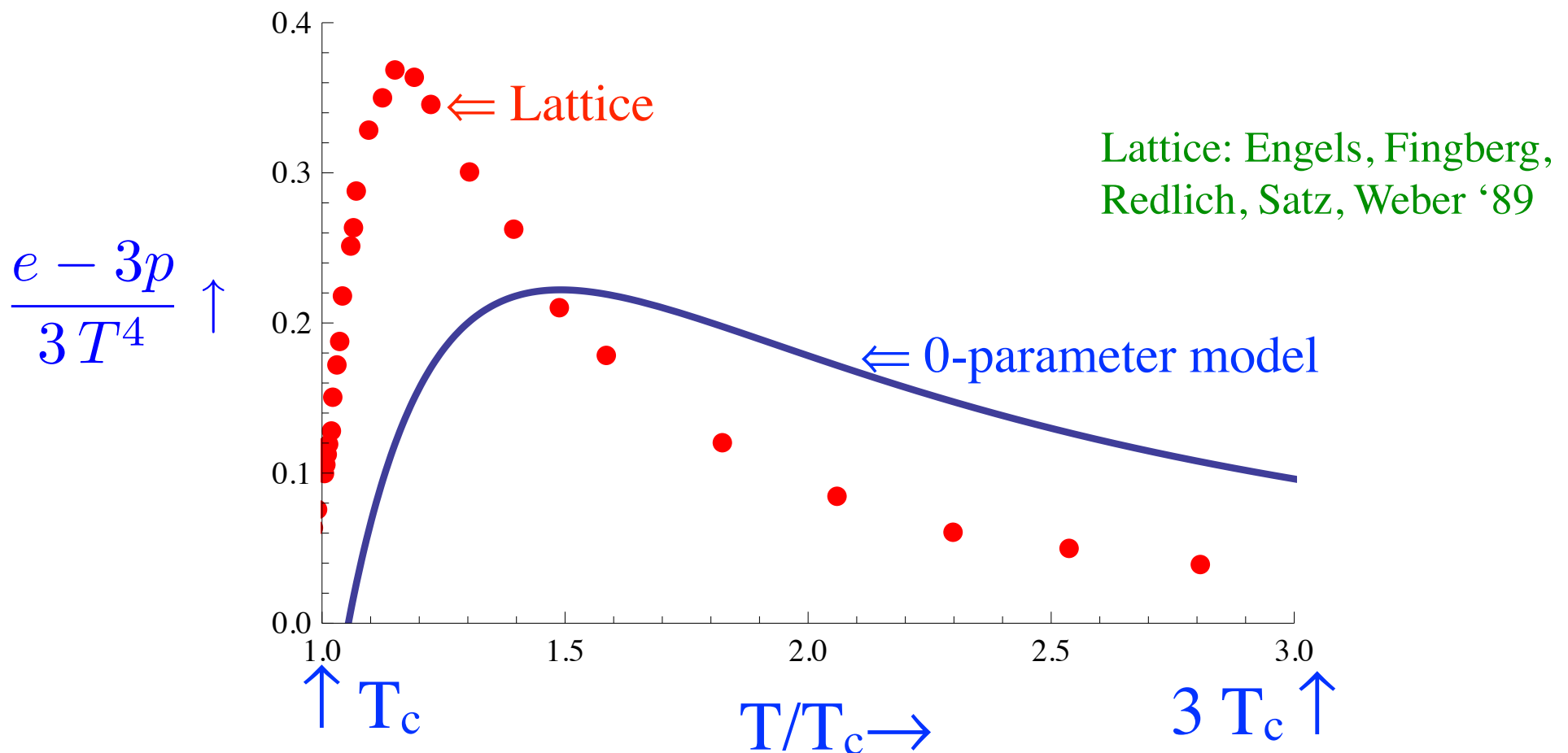
Meisinger, Miller, Ogilvie ph/0108009, MMO:

take $V_{\text{non}} \sim T^2$

$$V_{\text{non}}(q) = \frac{4\pi^2}{3} T^2 T_c^2 \left(-\frac{c_1}{5} q(1-q) + \frac{c_3}{15} \right)$$

Two conditions: transition occurs at T_c , pressure(T_c) = 0

Fixes c_1 and c_3 , *no* free parameters. *Not* close to lattice data (*from* '89!)



1-parameter matrix model, $N = 2$

Dumitru, Guo, Hidaka, Korthals-Altes, RDP '10: to usual perturbative potential,

$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \left(-\frac{1}{20} + q^2(1-q)^2 \right)$$

Add a non-perturbative potential $V_{non} \sim T^2 T_c^2$. Also add a term like that in V_{pert} :

$$V_{non}(q) = \frac{4\pi^2}{3} T^2 T_c^2 \left(-\frac{c_1}{5} q(1-q) - c_2 q^2(1-q)^2 + \frac{c_3}{15} \right)$$

Now just like any other mean field theory. $\langle q \rangle$ given by minimum of V_{eff} :

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q) \qquad \left. \frac{dV_{eff}(q)}{dq} \right|_{q=\langle q \rangle} = 0$$

$\langle q \rangle$ is (implicitly) T -dependent. Pressure:

$$p(T) = -V_{eff}(\langle q \rangle)$$

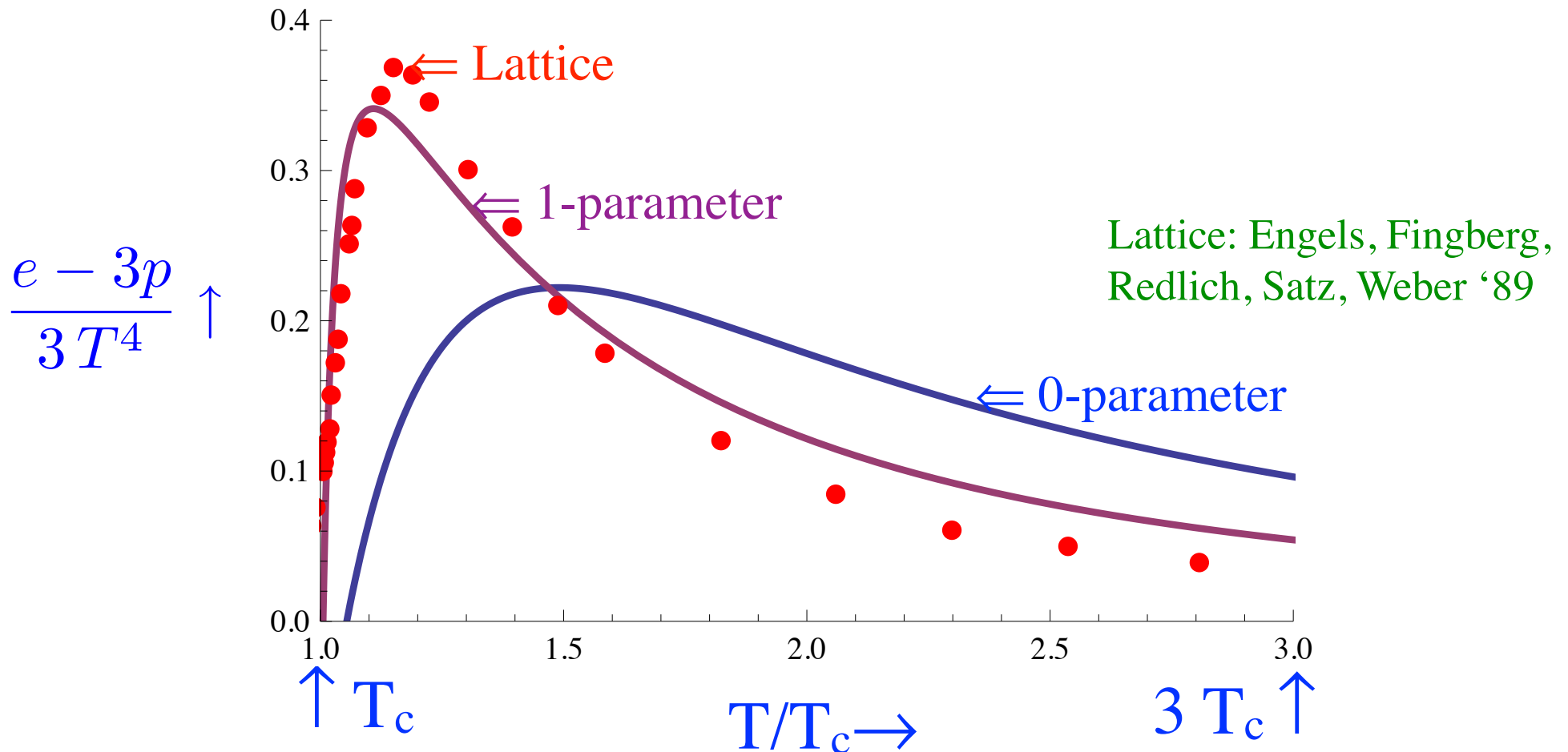
Lattice vs matrix models, $N = 2$

Choose c_2 to fit $e-3p/T^4$: optimal choice

$$c_1 = 0.23, c_2 = .91, c_3 = 1.11$$

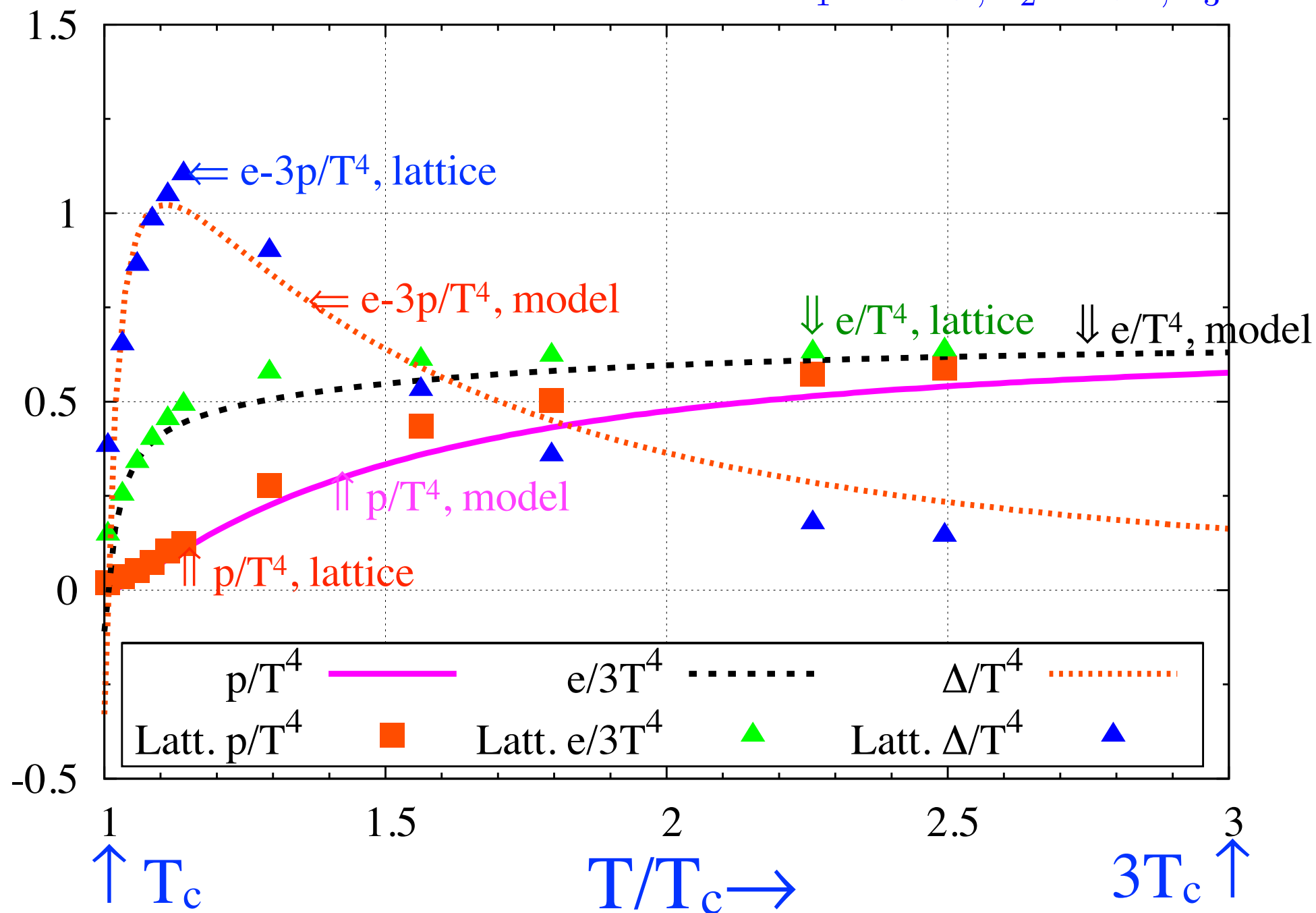
Reasonable fit to $e-3p/T^4$; also to p/T^4 , e/T^4 .

N.B.: $c_2 \sim 1$. At T_c , terms $\sim q^2(1-q)^2$ *almost* cancel.



Lattice vs 1-parameter model, N = 2

$$c_1 = 0.23, c_2 = .91, c_3 = 1.11$$



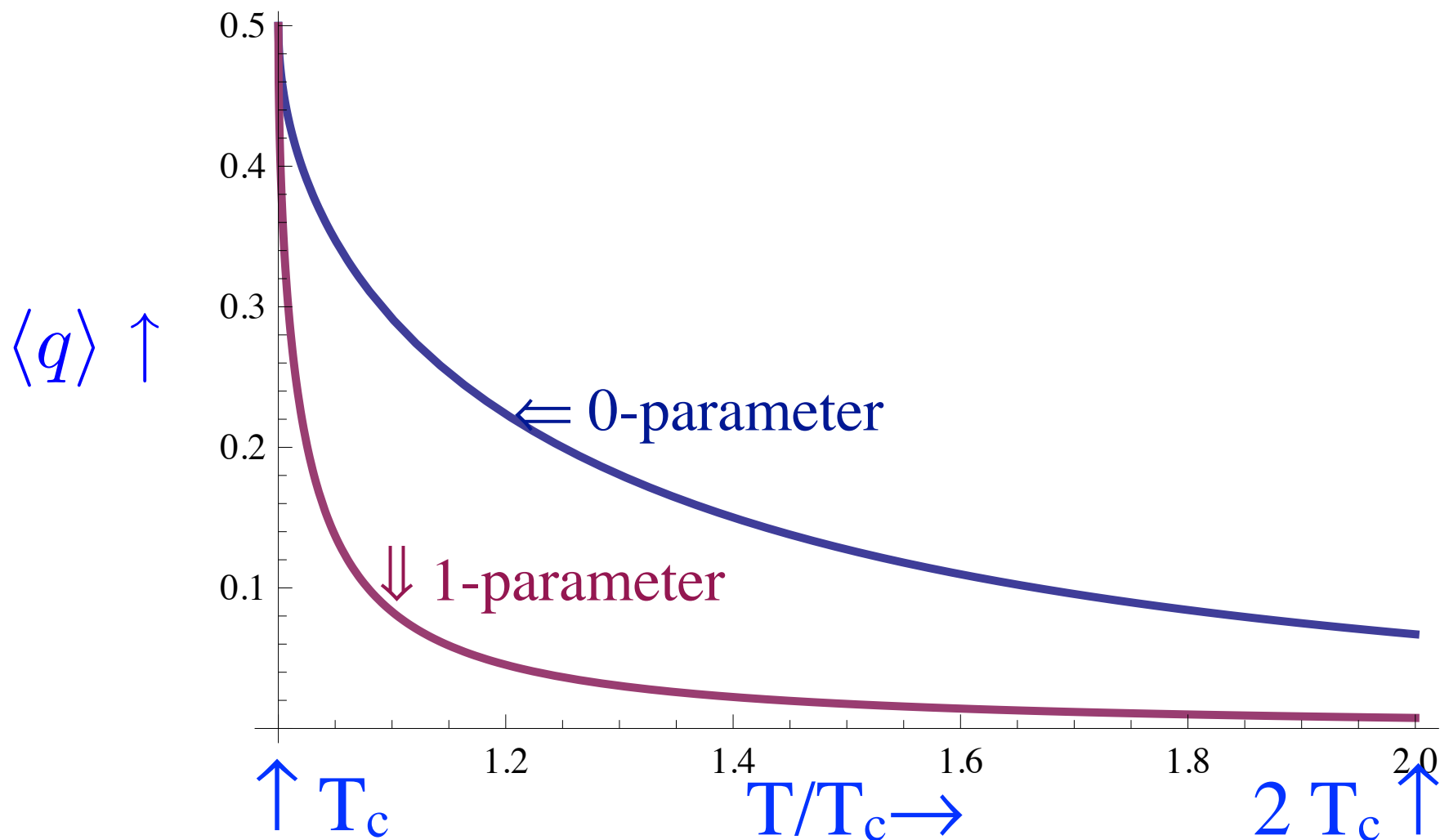
Width of transition region, 0- vs 1-parameter

1-parameter model: get sharper e^{-3p/T^4} because $\langle q \rangle \rightarrow 0$ *much* quicker above T_c .

Physically: sharp e^{-3p/T^4} implies region where $\langle q \rangle$ is significant is *narrow*

N.B.: $\langle q \rangle \neq 0$ at all T , but numerically, *negligible* above $\sim 1.2 T_c$; $p \sim \langle q \rangle^2$.

Above $1.2 T_c$, the T^2 term in the pressure is due *entirely* to the *constant* term, c_3 !

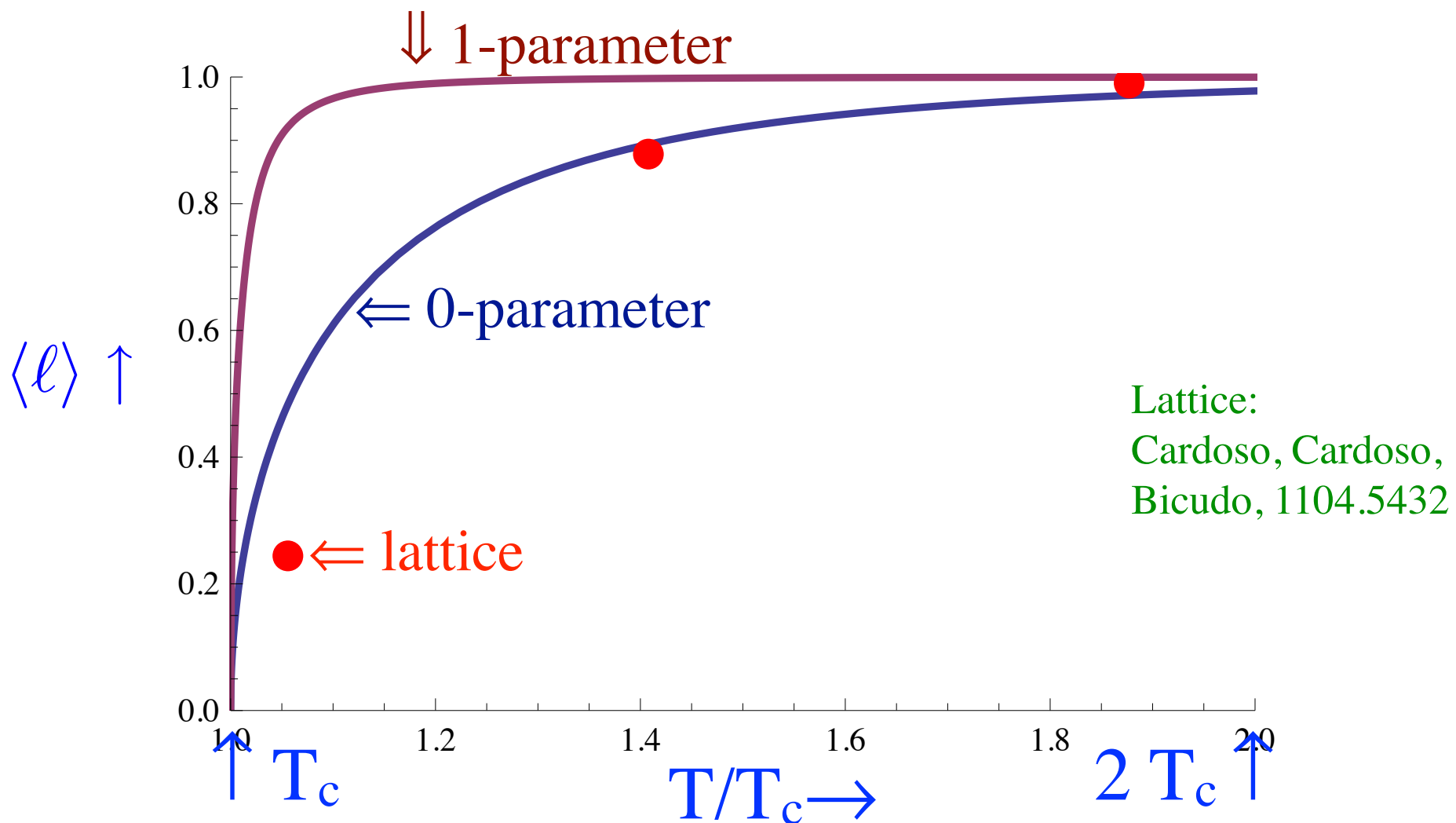


Polyakov loop: matrix models vs lattice, $N = 2$

Lattice: *renormalized* Polyakov loop. Matrix model: $\langle l \rangle = \cos(\pi q/2)$

0-parameter model: close to lattice

1-parameter model: *sharp* disagreement. $\langle l \rangle$ rises to ~ 1 *much* faster - ?



Interface tension, $N = 2$

σ vanishes as $T \rightarrow T_c$, $\sigma \sim (t-1)^{2\nu}$.

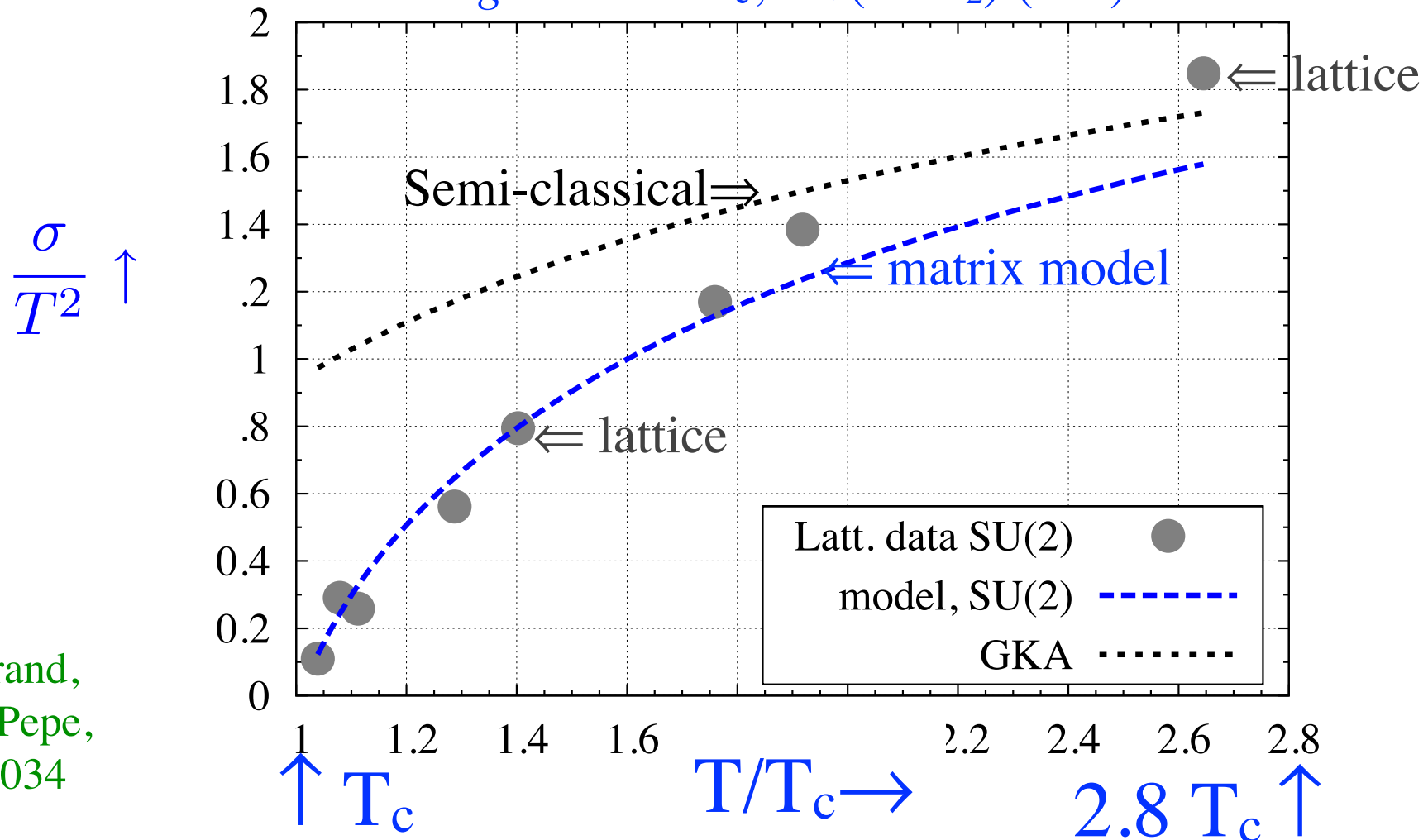
Ising $2\nu \sim 1.26$; Lattice: ~ 1.32 .

Matrix model: ~ 1.5 : c_2 important.

$$\sigma(T) = \frac{4\pi^2 T^2}{3\sqrt{6g^2}} \frac{(t^2 - 1)^{3/2}}{t(t^2 - c_2)}, \quad t = \frac{T}{T_c}$$

Semi-class.: **GKA '04**. Include corr.'s $\sim g^2$ in matrix $\sigma(T)$ (ok $T > 1.2 T_c$)

N.B.: width of interface *diverges* as $T \rightarrow T_c$, $\sim \sqrt{(t^2 - c_2)/(t^2 - 1)}$.



Lattice:
de Forcrand,
D'Elia, Pepe,
lat/0007034

Adjoint Higgs phase, $N = 2$

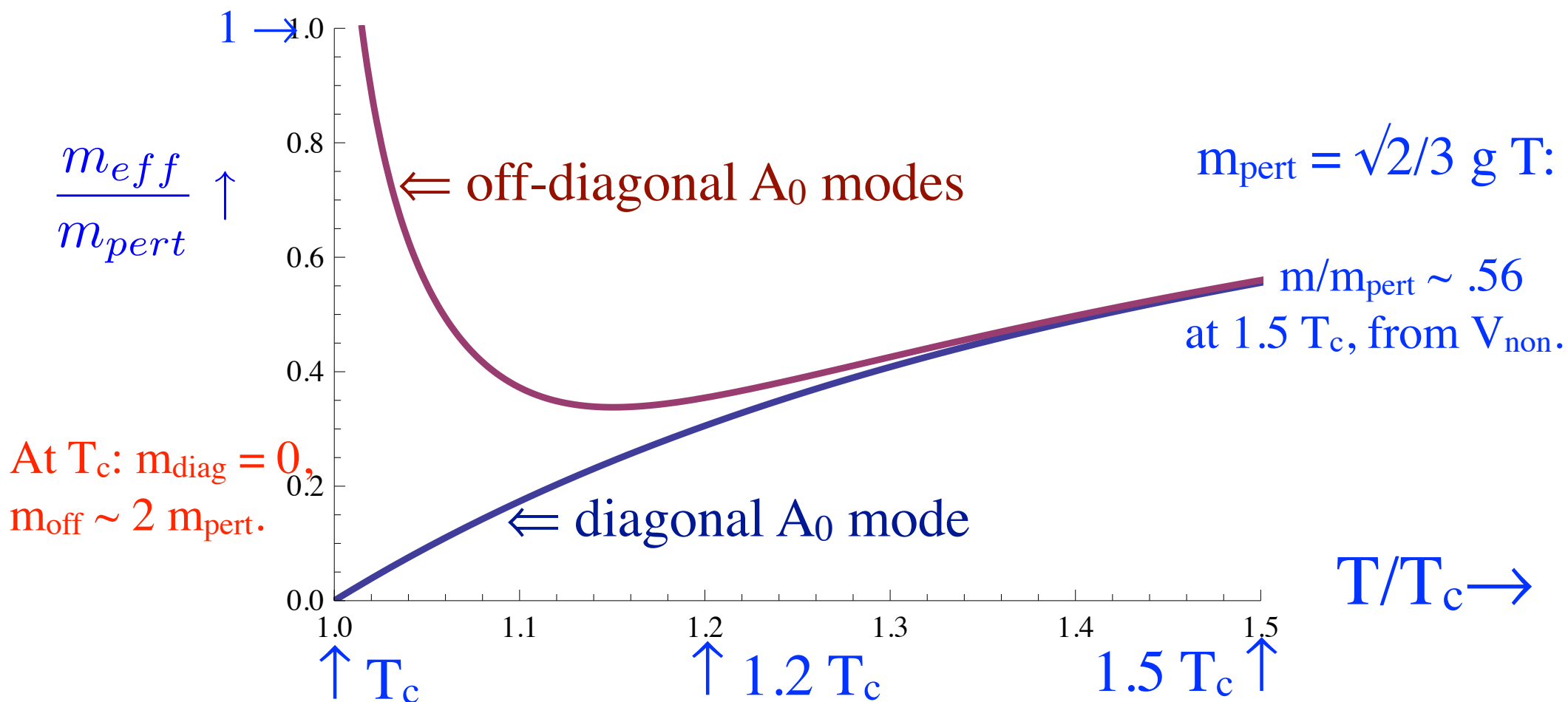
$A_0^{\text{cl}} \sim q \sigma_3$, so $\langle q \rangle \neq 0$ generates an (adjoint) Higgs phase:

RDP, ph/0608242; Unsal & Yaffe, 0803.0344, Simic & Unsal, 1010.5515

In background field, $A = A_0^{\text{cl}} + A^{\text{qu}}$: $D_0^{\text{cl}} A^{\text{qu}} = \partial_0 A^{\text{qu}} + i g [A_0^{\text{cl}}, A^{\text{qu}}]$

Fluctuations $\sim \sigma_3$ not Higgs, $\sim \sigma_{1,2}$ Higgsed, get mass $\sim 2 \pi T \langle q \rangle$

Hence when $\langle q \rangle \neq 0$, when $T < 1.2 T_c$, *splitting of masses*:



Matrix model: $N \geq 3$

Why the transition is *always* 1st order

One parameter model

Z(3) paths in SU(3) gauge

For SU(3), *two* diagonal generators,

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$A_0 = \frac{2\pi T}{3g} (q_8 \lambda_8 + q_3 \lambda_3)$$

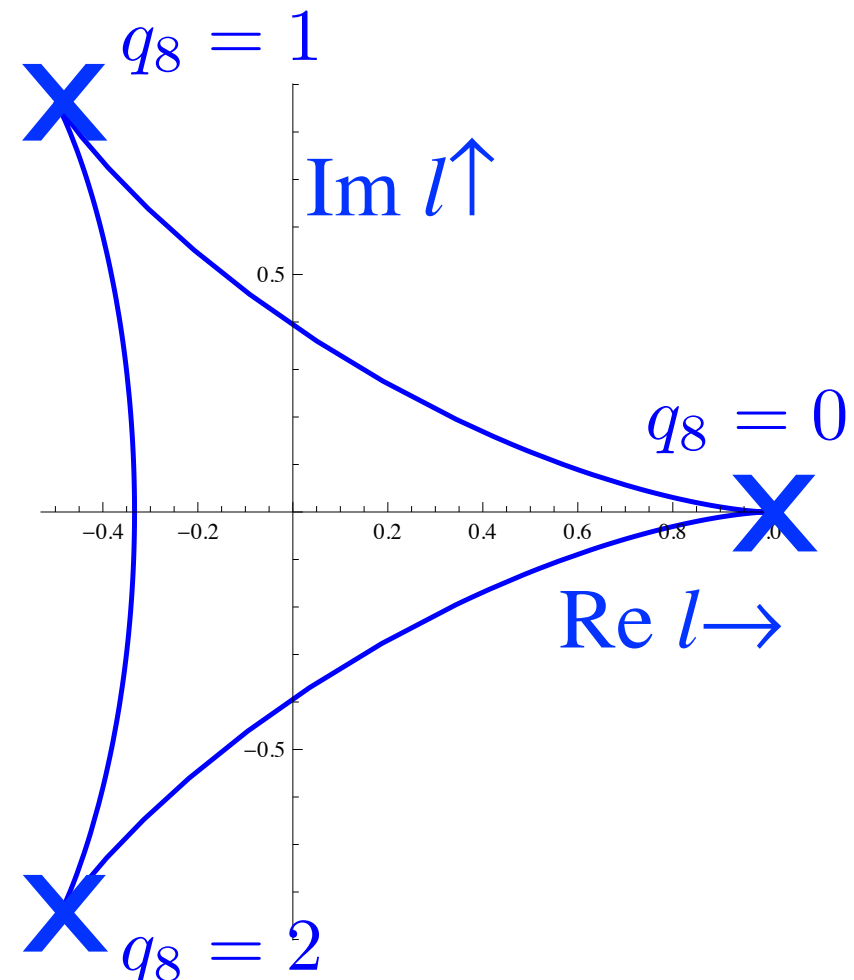
Z(3) paths: $q_8 \neq 0, q_3 = 0$:

$$\mathbf{L} = e^{2\pi i q_8 \lambda_8 / 3}$$

Three degenerate vacua, for $q_8 = 0, 1$, and 2 .

Move *between* vacua along blue lines,

$$\ell = \frac{1}{3} \text{tr } \mathbf{L} = \left(e^{2\pi i / 3} \right)^{q_8}, \text{ if } q_8 = 0, 1, 2$$



Confining vacuum in SU(3)

Alternately, consider moving along λ_3 .

In particular, consider $q_3 = 1$:

$$\mathbf{L} = e^{2\pi i q_3 \lambda_3 / 3}$$

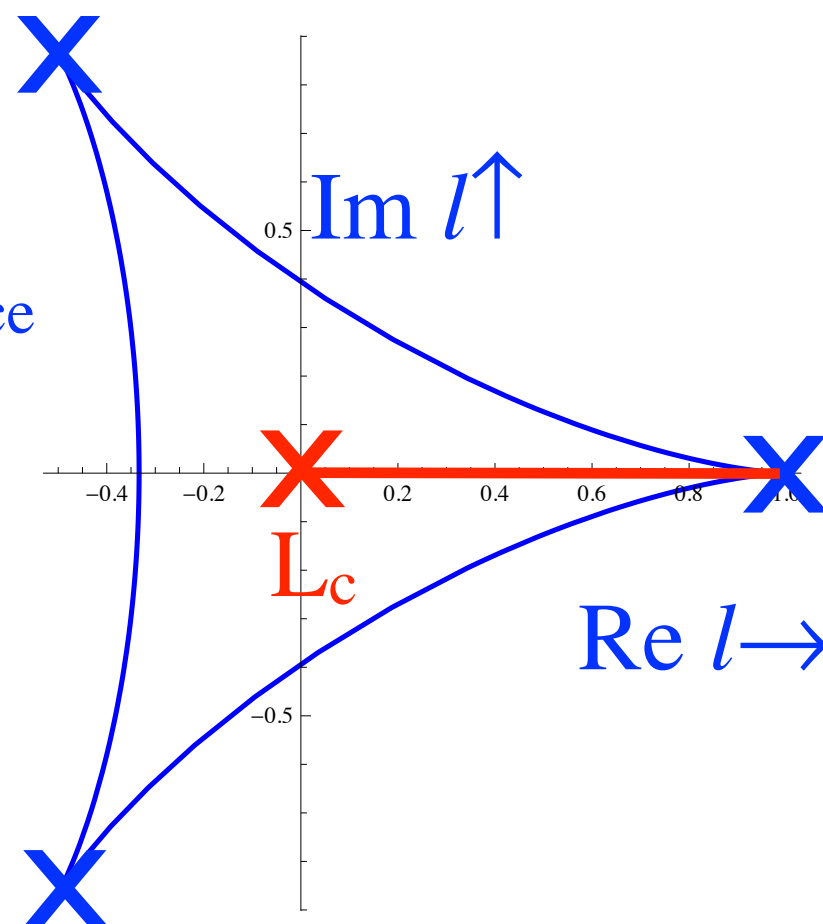
$$\mathbf{L}_c = \begin{pmatrix} e^{2\pi i/3} & 0 & 0 \\ 0 & e^{-2\pi i/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Elements of $e^{2\pi i/3} \mathbf{L}_c$ same as those of \mathbf{L}_c . Hence

$$\text{tr } \mathbf{L}_c = \text{tr } \mathbf{L}_c^2 = 0$$

\mathbf{L}_c is the confining vacuum, **X**:
“invariant” under $Z(3)$ transf.’s.

Move from deconfined vacuum, $\mathbf{L} = \mathbf{1}$,
to the confined vacua, \mathbf{L}_c , along red line:



Potentials for matrix models, any N

Simplest ansatz: constant, diagonal A_0 :

$$A_0^{ij} = \frac{2\pi T}{g} q_i \delta^{ij}, \quad i, j = 1 \dots N$$

At 1-loop order, perturbative potential

$$V_{pert}(q) = \frac{2\pi^2}{3} T^4 \left(-\frac{4}{15} (N^2 - 1) + \sum_{i,j} q_{ij}^2 (1 - q_{ij})^2 \right), \quad q_{ij} = |q_i - q_j|$$

Assume non-perturbative potential $\sim T^2 T_c^2$:

$$V_{non}(q) = \frac{2\pi^2}{3} T^2 T_c^2 \left(-\frac{c_1}{5} \sum_{i,j} q_{ij} (1 - q_{ij}) - c_2 \sum_{i,j} q_{ij}^2 (1 - q_{ij})^2 + \frac{4}{15} c_3 \right)$$

For $SU(N)$, $\sum_{j=1 \dots N} q_j = 0$. Hence $N-1$ independent q_j 's, # diagonal generators.

Getting to confinement in q_j space

For general problem of interfaces, need *all* $N-1$ directions in q_j space.

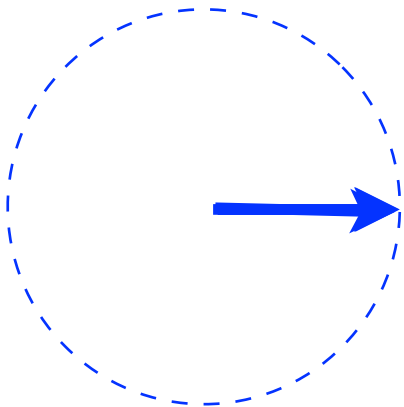
Move from pert. vacuum, to the confining point, along *one* direction, q_j^c :

Perturbative vacuum: $q = 0$;
Confining point: $q = 1/2$.

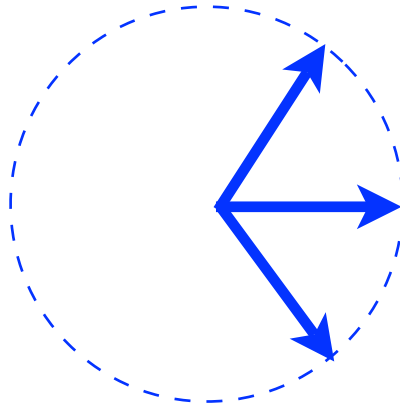
$$q_j^c = \left(\frac{2j - N - 1}{N} \right) q, \quad j = 1 \dots N$$

For 3 colors, 3 eigenvalues, $e^{\pm 2\pi i q/3}$, 1:

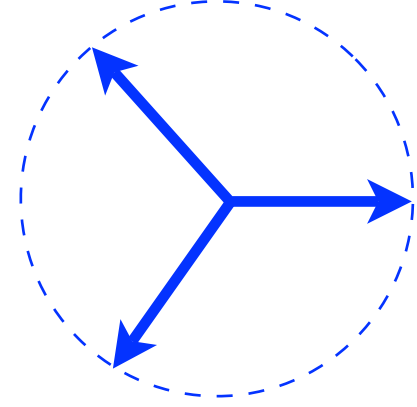
$$\ell = \frac{1}{3} \left(e^{2\pi i q/3} + e^{-2\pi i q/3} + 1 \right)$$



$$\langle \ell \rangle = 1$$



$$1 > \langle \ell \rangle > 0$$



$$\langle \ell \rangle = 0$$

Why deconfinement is 1st order for *all* $N \geq 3$

Define $\phi = 1 - 2q$,
Confining point $\phi = 0$

$$V_{tot} = \frac{\pi^2(N^2 - 1)}{45} T_c^4 t^2 (t^2 - 1) \tilde{V}(\phi, t), \quad t = \frac{T}{T_c}$$

$$\tilde{V}(\phi, t) = -m_\phi^2 \phi^2 - 2 \left(\frac{N^2 - 4}{N^2} \right) \phi^3 + \left(2 - \frac{3}{N^2} \right) \phi^4$$

$$m_\phi^2 = 1 + \frac{6}{N^2} - \frac{c_1}{t^2 - c_2}$$

Never a term linear in ϕ . No cubic term when $N = 2$, but for *any* $N \geq 3$!

The coefficient of the cubic term is special to the model, but its existence is *not*. Along q^c in q_j space, about $\phi = 0$ there is no symmetry of $\phi \rightarrow -\phi$ for *any* $N \geq 3$. Hence cubic terms in ϕ , and so a 1st order transition, are *generic*.

Special to matrix model, with the q_i 's elements of Lie *algebra*.

Svetitsky and Yaffe '80: from loops, elements of Lie *group*, 1st order *only* for $N = 3$

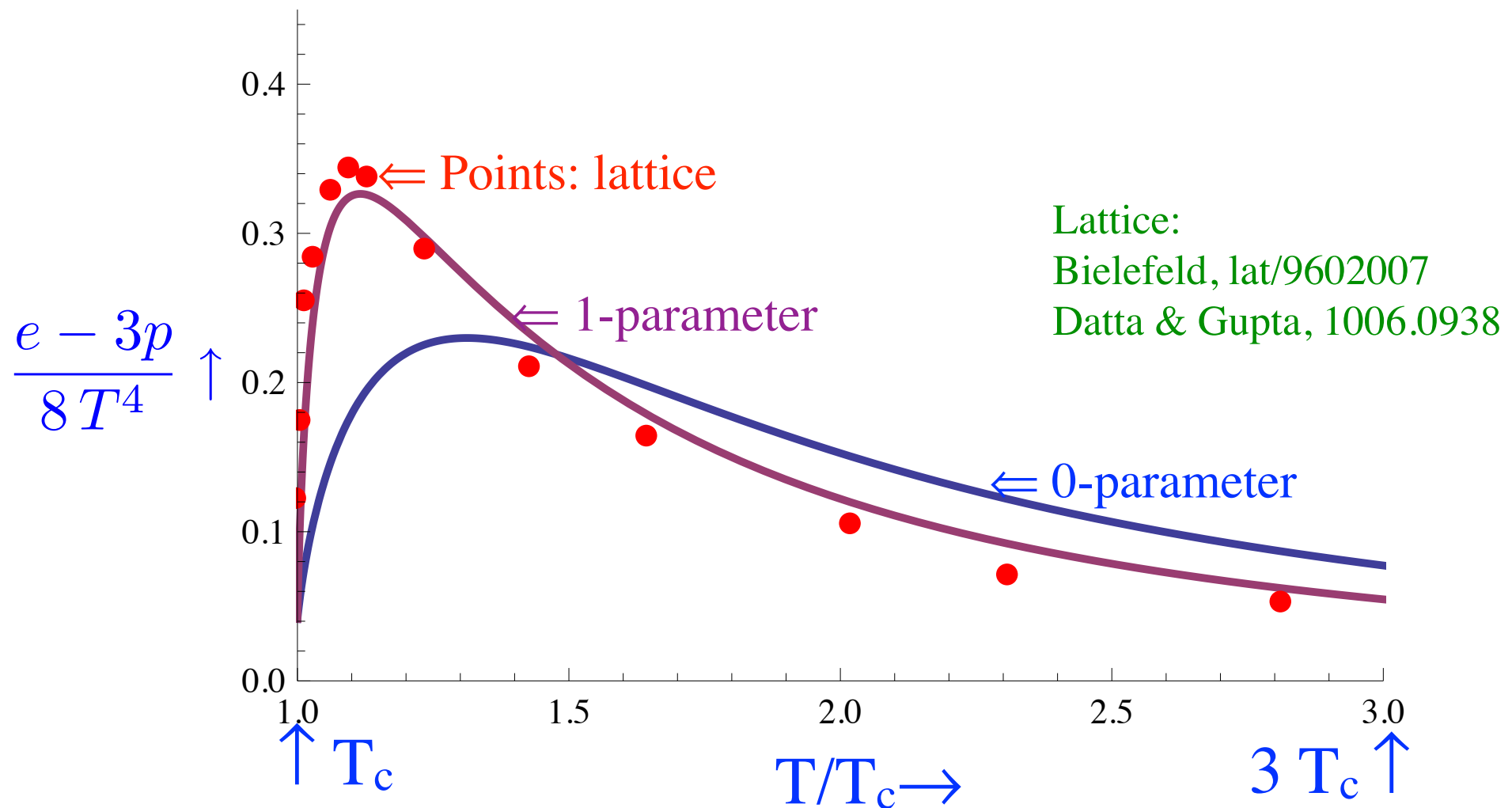
Lattice vs 0- and 1- parameter matrix models, $N = 3$

Results for $N=3$ similar to $N=2$.

0-parameter model way off.

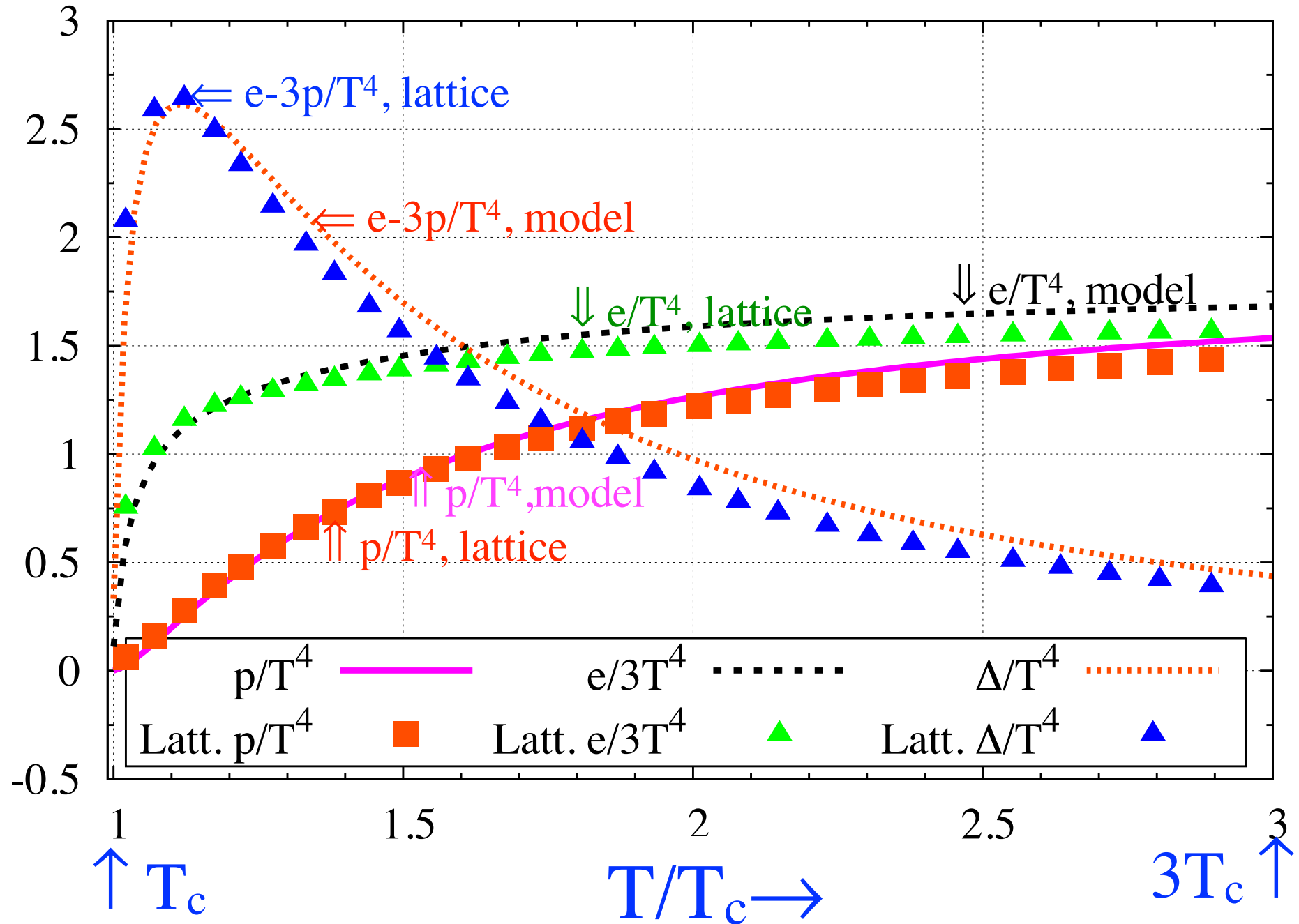
Good fit $e-3p/T^4$ for 1-parameter model, $c_1 = 0.32, c_2 = 0.83, c_3 = 1.13$

Again, $c_2 \sim 1$, so at T_c , terms $\sim q^2(1-q)^2$ *almost* cancel.



Lattice vs 1- parameter model, N = 3

$$c_1 = 0.32, c_2 = 0.83, c_3 = 1.13$$



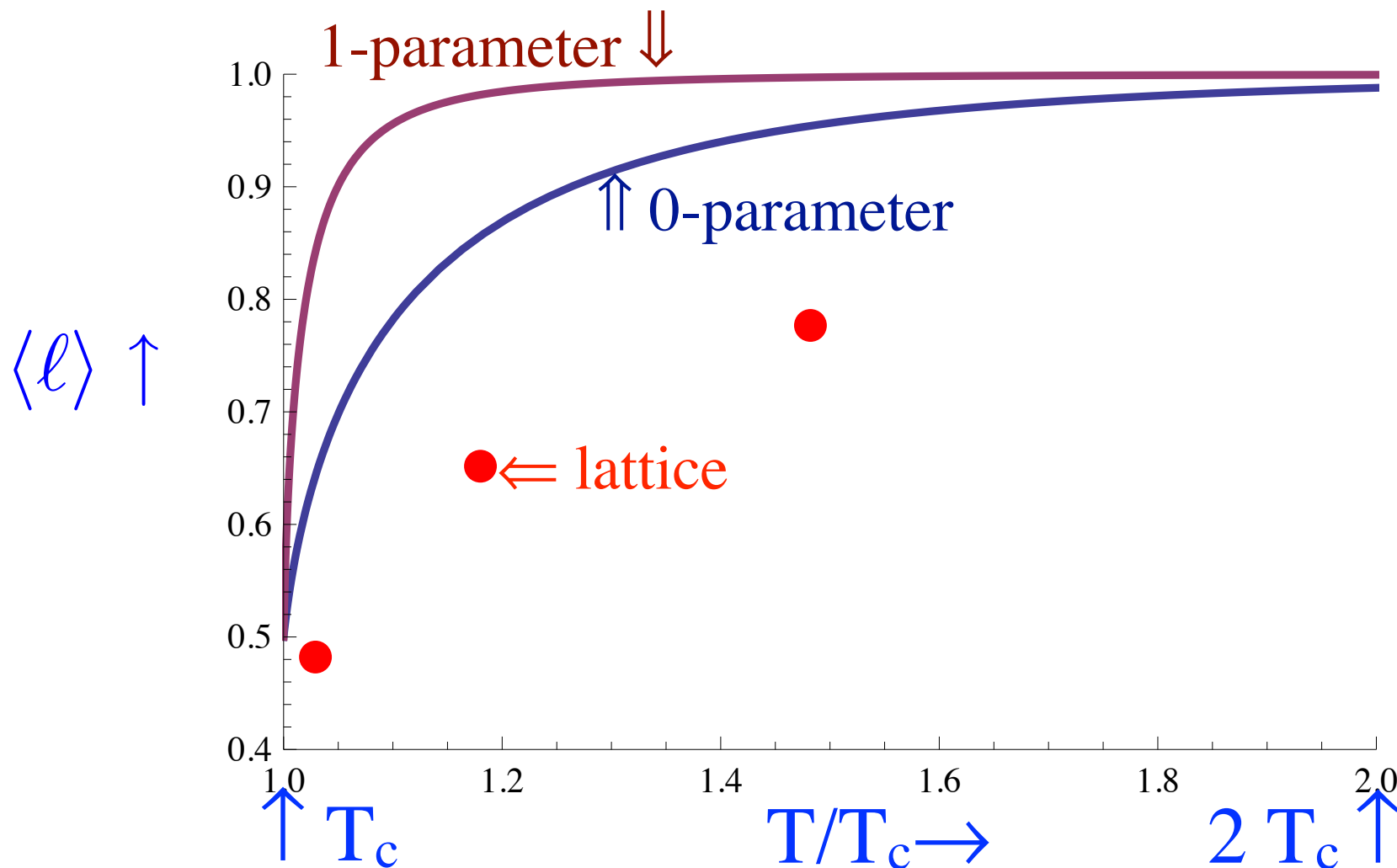
Polyakov loop: matrix models vs lattice, $N = 3$

Renormalized Polyakov loop from lattice does *not* agree with *either* matrix model

$\langle l \rangle - 1 \sim \langle q \rangle^2$: sharper e^{-3p/T^4} for 1-parameter model due to quicker decrease in $\langle q \rangle$

By $1.2 T_c$, $\langle q \rangle$ negligible, $\sim .05$

Again, for $T > 1.2 T_c$, the T^2 term in pressure due *entirely* to the *constant* term, c_3 !

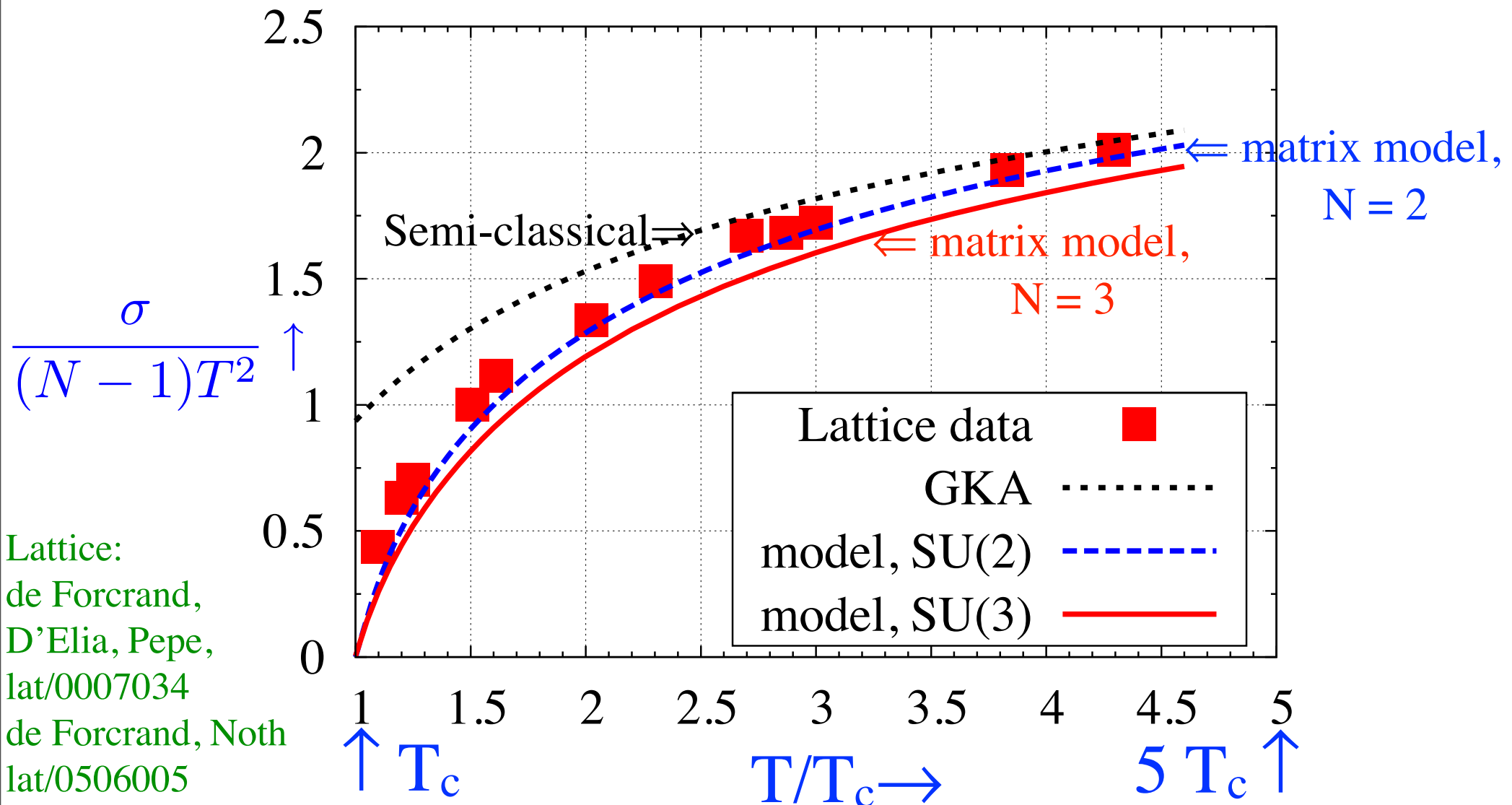


Interface tension, $N = 2$ and 3

Order-order interface tension, σ , from matrix model close to lattice.

For $T > 1.2 T_c$, path along λ_8 ; for $T < 1.2 T_c$, along *both* λ_8 and λ_3 .

$\sigma(T_c)/T_c^2$ nonzero but *small*, $\sim .02$. Results for $N=2$ and $N=3$ similar - ?

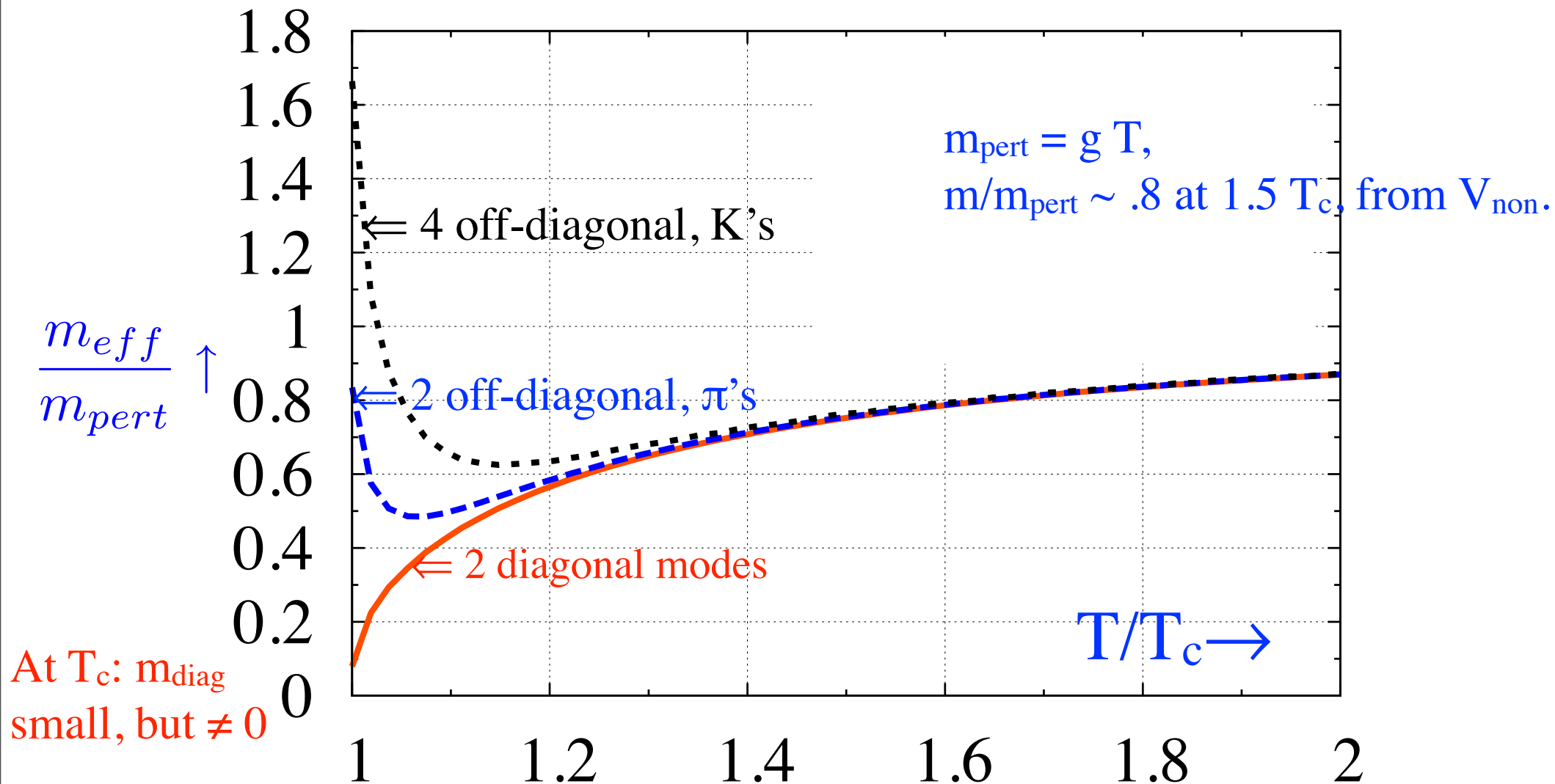


Adjoint Higgs phase, $N = 3$

For $SU(3)$, deconfinement along $A_0^{cl} \sim q \lambda_3$. Masses $\sim [\lambda_3, \lambda_i]$: two off-diagonal.

Splitting of masses only for $T < 1.2 T_c$:

Measureable from singlet potential, $\langle \text{tr } L^\dagger(x) L(0) \rangle$, over *all* x .



Matrix model: $N \geq 3$

To get the latent heat right,

Two parameter model.

Improve thermodynamics, interface tensions.

Latent heat, and a 2-parameter model

Latent heat, $e(T_c)/T_c^4$: 1-parameter model too small:

1-para.: 0.33. **BPK**: $1.40 \pm .1$; **DG**: $1.67 \pm .1$.

$$c_3(T) = c_3(\infty) + \frac{c_3(1) - c_3(\infty)}{(T/T_c)^2}$$

2-parameter model, $c_3(T)$. Like MIT bag constant

WHOT: $c_3(\infty) \sim 1$. *Fit* $c_3(1)$ to DG latent heat

$$c_3(1) = 1.33, c_3(\infty) = .95$$

Fits lattice for $T < 1.2 T_c$, overshoots above.

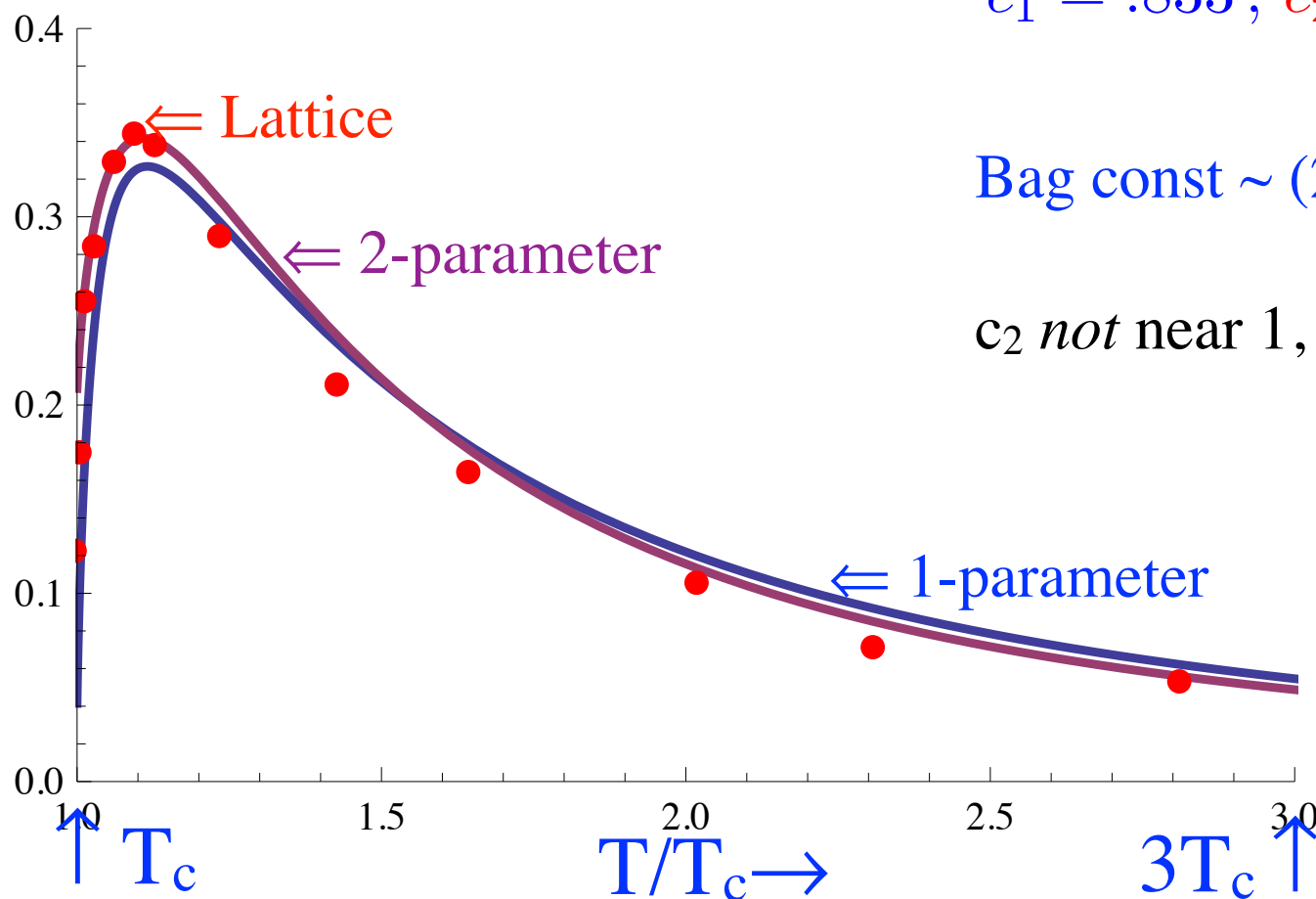
$$c_1 = .833, c_2 = .552$$

$$\text{Bag const} \sim (203 \text{ MeV})^4$$

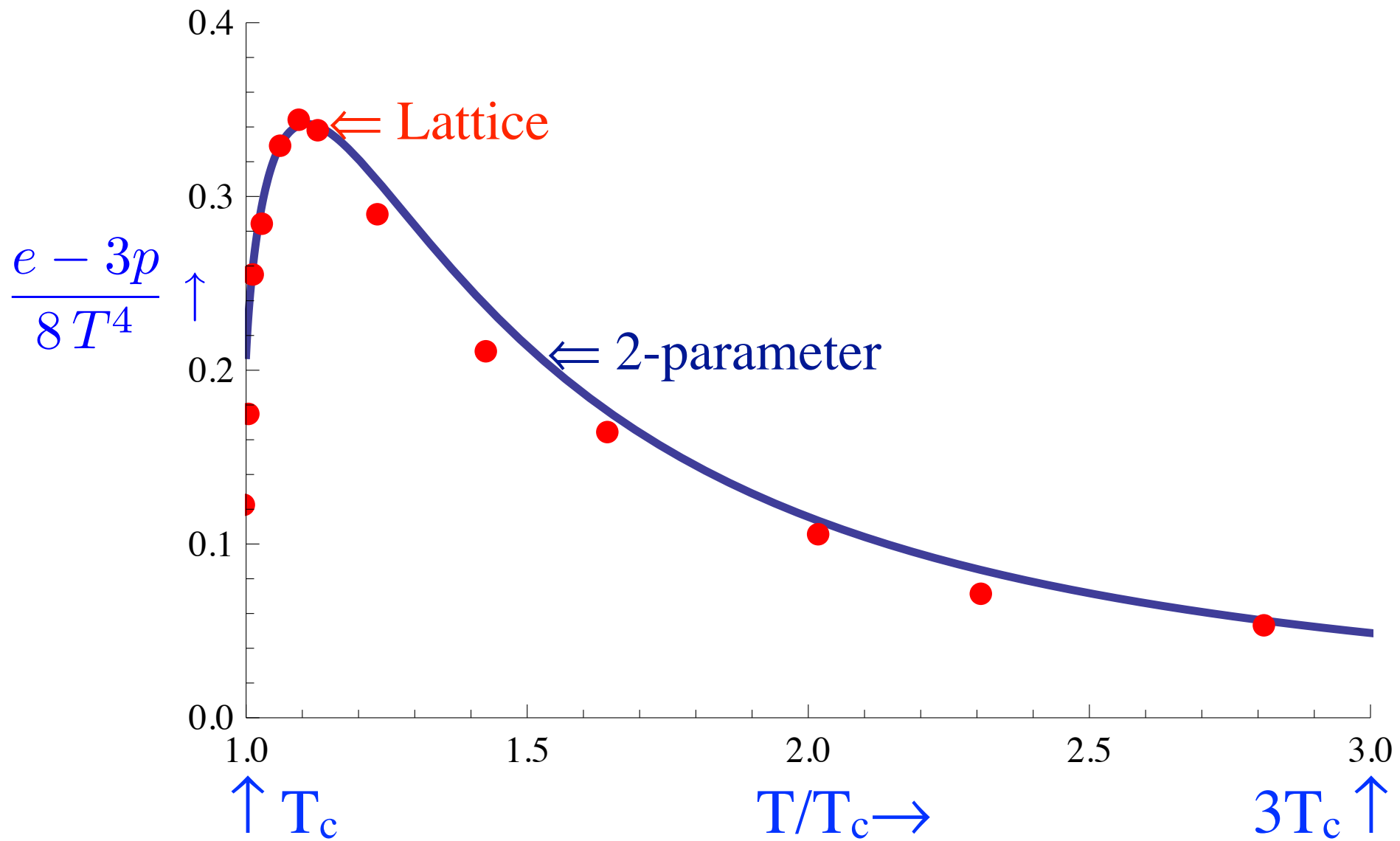
c_2 *not* near 1, vs 1-para.

$$\frac{e - 3p}{8 T^4} \uparrow$$

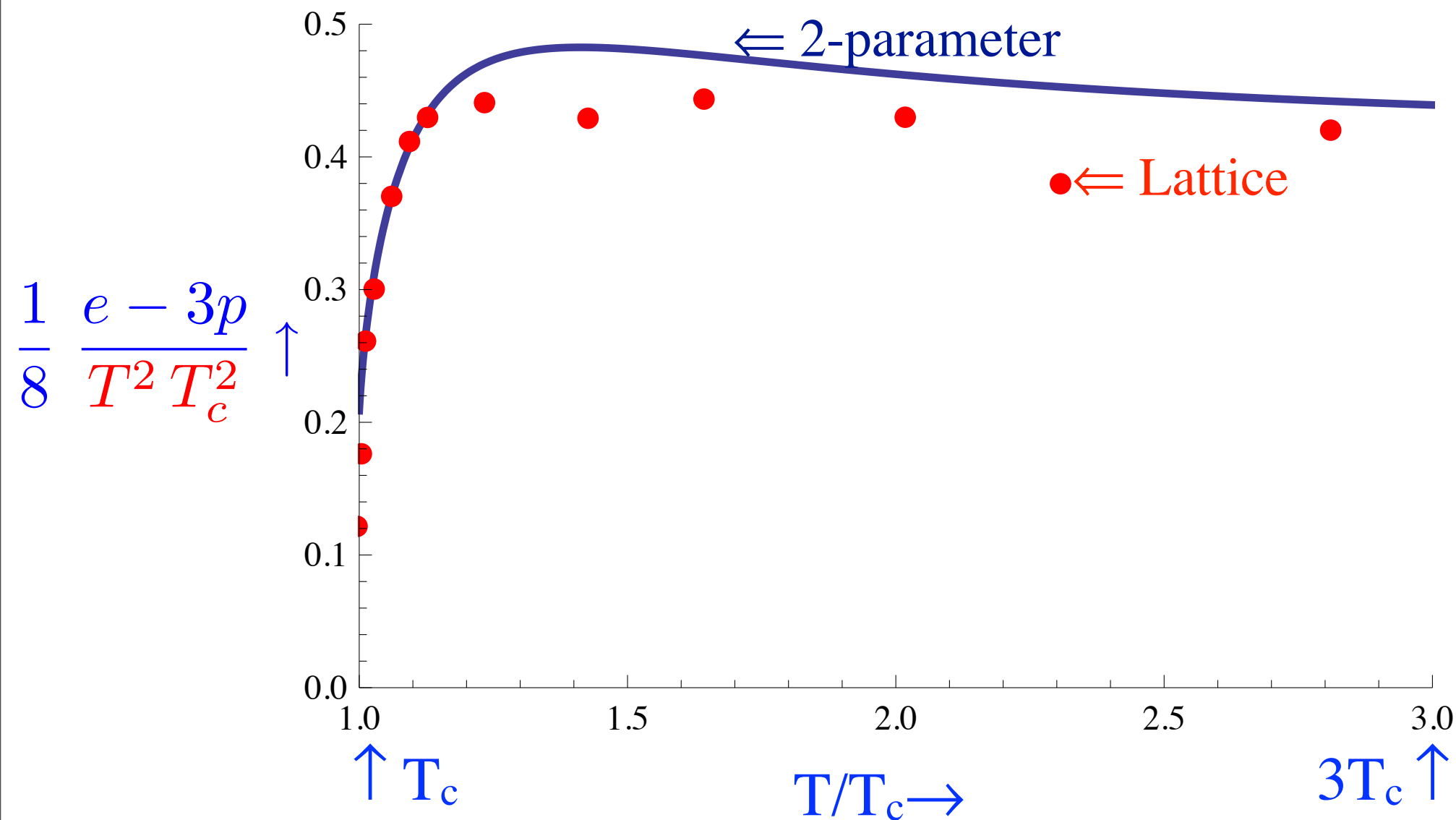
Latent heat, lattice:
BPK: Beinlich,
Peikert, Karsch
lat/9608141
DG: Datta, Gupta
1006.0938



Anomaly: 2-parameter model vs lattice

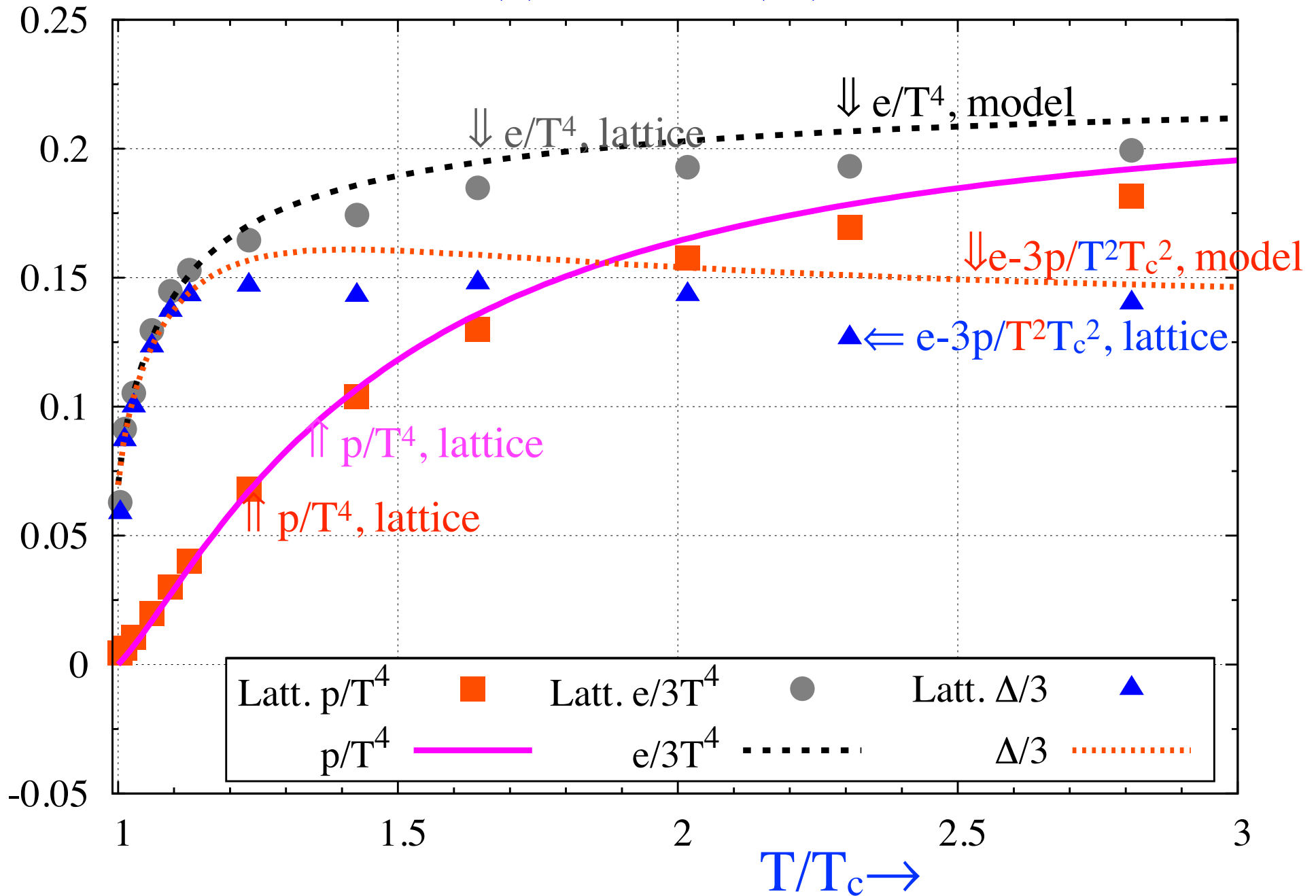


Anomaly times T^2 : 2-parameter model vs lattice



Thermodynamics of 2-parameter model, $N = 3$

$$c_3(1) = 1.33, c_3(\infty) = .95, c_1 = .833, c_2 = .552$$

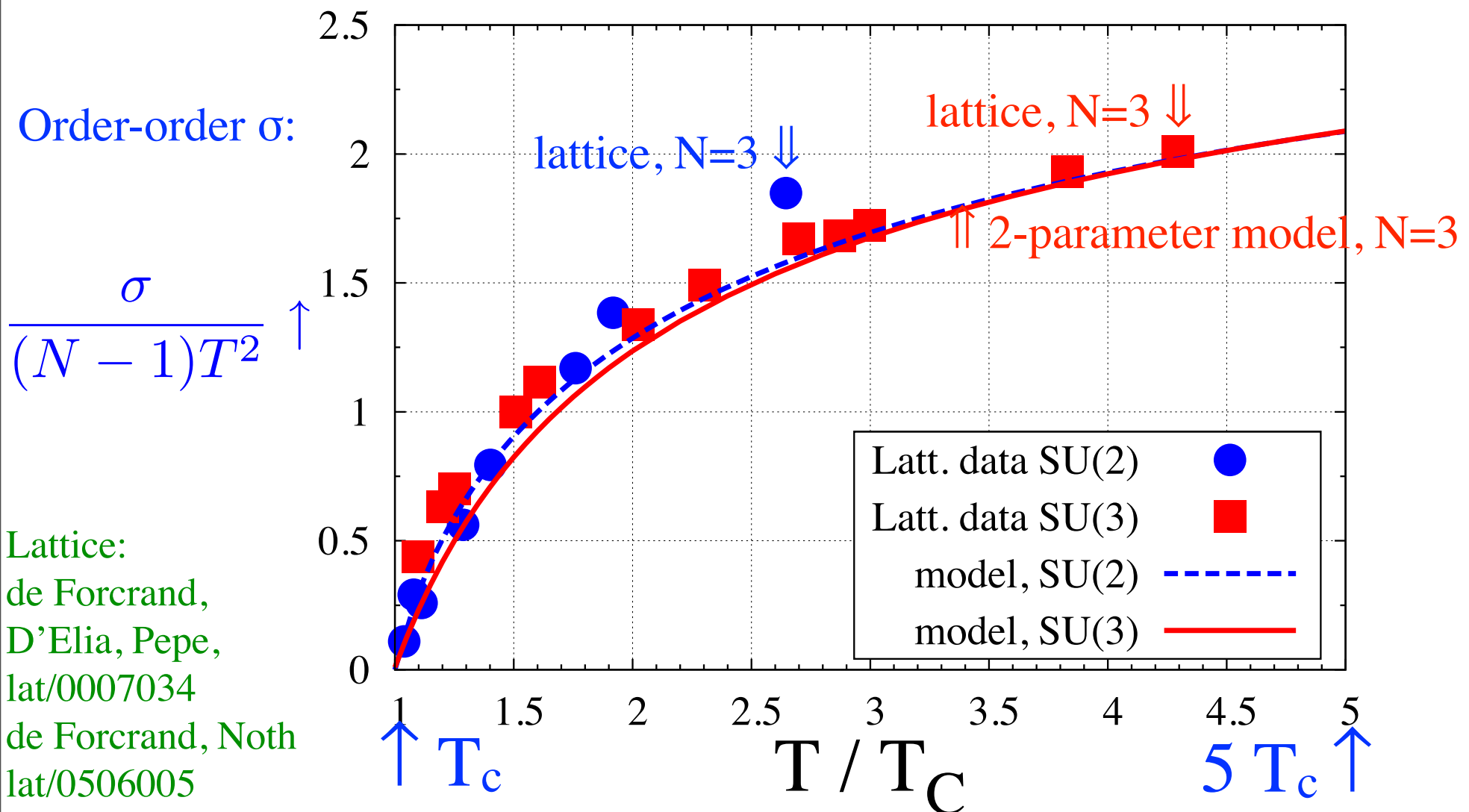


Interface tensions, 2-parameter model, $N = 3$

Order-order interface tension, σ , close to lattice. Order-order $\sigma(T_c)/T_c^2 \sim .043$.

1st order transition, so can compute order-disorder $\sigma(T_c)/T_c^2 \sim .022$, vs

Lattice: Lucini, Teper, Wegner, lat/0502003, .019 Beinlich, Peikert, Karsch lat/9608141 0.16

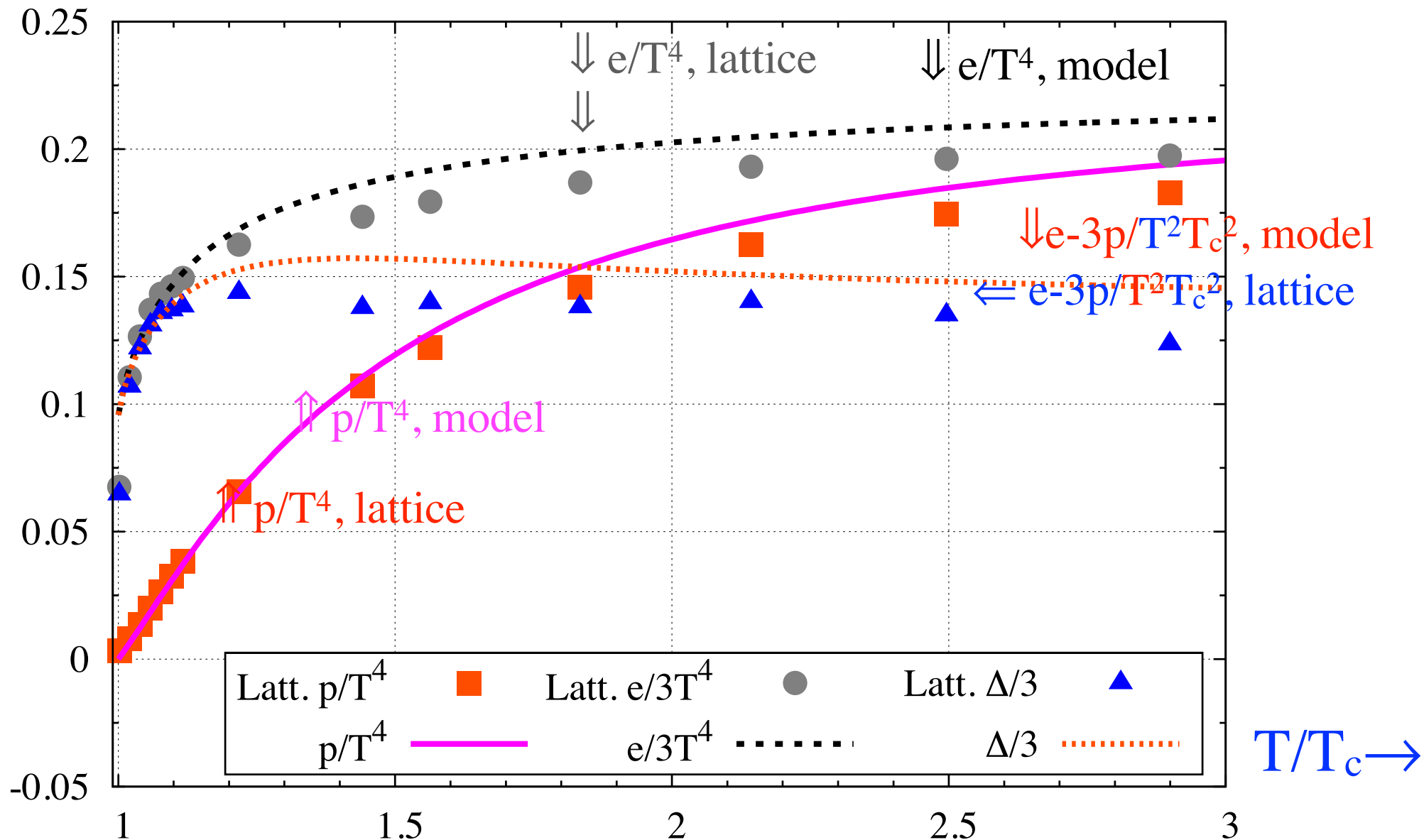


2-parameter model, $N = 4$

Assume $c_3(\infty) = 0.95$, like $N=3$. Fit $c_3(1)$ to latent heat, Datta & Gupta, 1006.0938

Order-disorder $\sigma(T_c)/T_c^2 \sim .08$, vs lattice, .12, Lucini, Teper, Wegner, lat/0502003

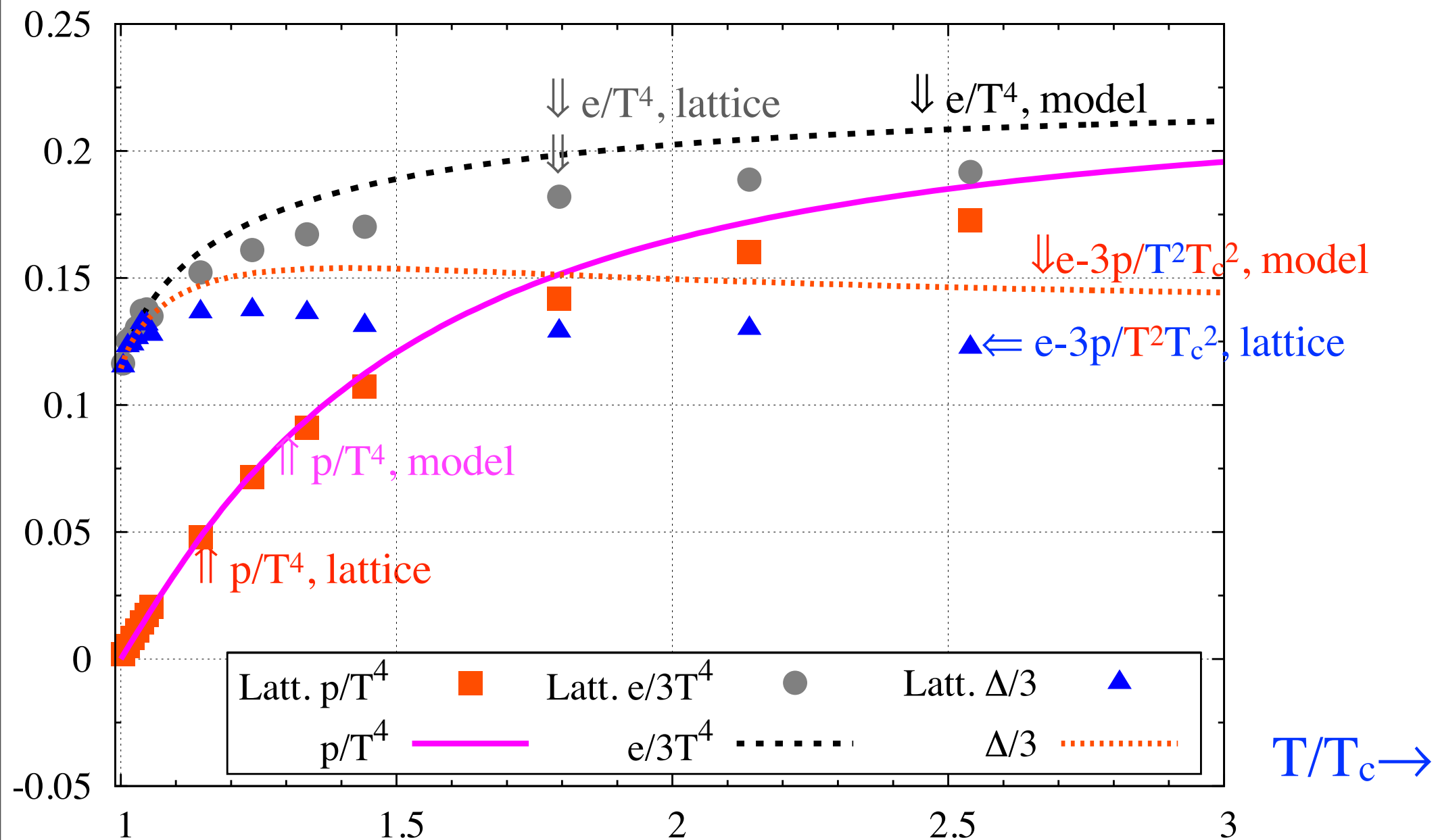
$$c_3(1) = 1.38, c_3(\infty) = .95, c_1 = 1.025, c_2 = 0.39$$



2-parameter model, $N = 6$

Order-disorder $\sigma(T_c)/T_c^2 \sim .25$, vs lattice, .39, Lucini, Teper, Wegner, lat/0502003

$$c_3(1) = 1.42, c_3(\infty) = .95, c_1 = 1.21, c_2 = 0.23$$



Conclusions

Transition region *narrow*: for pressure, $< 1.2 T_c$!

For interface tensions, $< 4 T_c$...

Above $1.2 T_c$, pressure dominated by *constant* term $\sim T^2$.

What does this term come from? Gluon mass (for spatial gluons)?

In 2+1 dimensions, ideal T^3 . Caselle + ...: also T^2 term in pressure.

But mass would be $m^2 T$, not $m T^2$.

T^2 term like free energy of massless fields in 2 dimensions: string? Above T_c ?

To say the least: need to include quarks!

Can then compute temperature dependence of:

shear viscosity, energy loss of light quarks, quarkonia damping....

Lattice: SU(N) in 2+1 dimensions

Caselle, Castagnini, Geo, Gliozzi, Panero, 1105.0359, unpublished

SU(N) for N = 2, 3, 4, 5

$$p(T) \approx \# T^2 (T - c T_c), \quad c \approx 1.$$

